Lecture Notes: Chapter 14

Waves and Sound

Notes by A. W. Watson for La Salle University, Spring 2016.
[This is a 2-day lecture.]
Outline taken from pages 488-490 of Walker's Physics, 4th. Ed. (2014).

Last chapter, we discussed oscillators (masses attached to springs, pendula, etc.). By themselves, these are interesting physical systems. But if you attach lots of these oscillators together one after another, or just put a bunch of them close together in the same general area, you can get even more interesting phenomena. Like people standing up and sitting down to do "the wave" at sporting events, individual oscillations can work together to form waves.

[This is an example of a microstate (a single person) and a macrostate ("the wave"). The behavior of the microstate may be slightly, or even completely different, from the macrostate. Microstates and macrostates are very important in subjects like statistical physics and thermodynamics.]

14.1 Types of Waves

Think for a second about "the wave".

What happens during it?

I stand up, lift my arms up, then put my arms down, and sit down. The person next to me does the same thing, but starts a second or two later than I do. Then the person next to him does the same thing, but a second or two later than he. Etc.

We can "zoom in" on this motion and look at a single person, moving up and down like a single oscillator, or we can "zoom out" and look at the crowd as a whole, where it looks like a "bump" is moving through the stands – that's "the wave" itself. Putting this in more rigid terms, we can say that a wave is a propagating disturbance.

In any wave-like motion, the individual particles or oscillators only move a tiny bit, much less than the wavelength of the wave. Physical waves need a *medium* to travel through – whether it's solid, or liquid, or gaseous. The molecules and atoms that make up these materials can wiggle in two general directions – perpendicular (at right angles) to the motion of the wave, or parallel to it (in the same direction). This lets us define two categories of physical waves:

Transverse Waves and Longitudinal Waves

In a transverse wave individual particles move at right angles to the direction of wave propagation. In a longitudinal wave individual particles move in the same direction as the wave propagation.

A good example of a transverse wave is a wave on a string (see Figure 14-1 on p 453); a good example of a longitudinal wave is a wave on a compressed Slinky. Ocean waves are transverse; sound waves are longitudinal. But both kinds of waves can be modeled mathematically as sinusoidal curves.

[Draw sine wave on the board and label wavelength λ, amplitude A, crest, trough; and explain relationship between speed v, frequency f, and wavelength λ.]

Wavelength, Frequency, and Speed

The wavelength, λ, frequency, f, and speed, v, of a wave are related by

$$v = \lambda f \tag{14-1}$$

[Do Exercise 14-1 on p 455. Maybe make one up with light (400 nm)?]

14.2 Waves on a String

The speed of a wave travelling down a string depends on two things: the tension in the string, and the mass per unit length of the string. Let's start with the tension. Transverse waves can propagate on a string held taut with a tension force, F. Consider the alternative: a string with no net tension applied. If you string a guitar, but don't tighten the strings and try to play it, no waves will resonate on the strings.

But if you tighten all the strings equally, you'll notice that the thinner strings are higher-pitched. That's because the waves along those strings move more quickly (increasing v while keeping λ the same increases the frequency f), increasing the frequency. From this, we can conclude that the speed of a wave through a string is dependent on the relative "thickness" of a string, which we can express mathematically as its mass per unit length:

Mass per Length

The mass per length of a string is $\mu = m/L$.

There is a good explanation on page 456 **[which I encourage you to read]** of how we arrive at this formula, but combining what we've just said, we can conclude that:

Speed of a wave on a String

The speed of a wave on a string with a tension force F and a mass per unit length μ is

$$v = \sqrt{\frac{F}{\mu}} \tag{14-2}$$

The last important thing to note here is that, as can be seen in Figures 14-7 and 14-8, waves reflect differently, depending on whether or not the end is fixed.

Can you explain why this is?

Read "Reflections" on p 458 and try to answer.

[Do Example 14-1 on page 457, and read the Insight.]

Reflections

If the end of a string is fixed, the reflection of a wave is inverted. If the end of a string is free to move transversely, waves are reflected with no inversion.

14.3 Harmonic Wave Functions

A harmonic wave has the shape of a sine or a cosine.The textbook gives a derivation of the following formula on page 459, but we won't go in to it here:

Wave Function

A harmonic wave of wavelength λ and period T is described by the following

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \tag{14-4}$$

One thing that you might notice is weird is that, here, y is a function of *two* variables, x, and t. But these are just sine curves (on p 459), which we know we can describe with only one dependent variable, like $y(t)$.

So how can we remove the x-dependence from $y(x,t)$?

Substitute $x' = vt'$, then $f = 1/T$, then $f = v/\lambda$. Then...

$$y(vt', t) = A\cos\left(2\pi f t' - 2\pi f t\right)$$

Now y only cares about the final and initial times, and some constants which describe the wave

$$y(t', t) = A\cos\left(\omega(t'-t)\right) = A\cos\left(\omega \Delta t\right)$$

Since $\omega = \Delta\theta/\Delta t$, we could also write:

$$y = A\cos\left(\Delta\theta\right)$$

Where y is now completely independent of both x and t. How would you interpret this? (Look at p 459 – $\Delta\theta$ can be thought of as a "phase shift from the "standard" solution, where $y(0, 0) = A$.)

14.4 Sound Waves

As noted earlier, sound is an example of a compressional, or longitudinal wave. A sound wave is a longitudinal wave of compressions and rarefactions that can travel through the air, as well as through other gases, liquids, and solids. Waves travel more quickly through solids than through gases, but we're most familiar with the speed of sound in air.

Speed of Sound

The speed of sound in air, under typical conditions, is $v = 343$ m/s or 770 mph.

Frequency of Sound

The frequency of sound determines its pitch. High-pitched sounds have high frequencies; low-pitched sounds have low frequencies.

Human Hearing Range

Human hearing extends from 20 Hz to $20,000$ Hz.

[Do Example 14-2 on p 461.]

14.5 Sound Intensity

The loudness of a sound is determined by its intensity.

Intensity

Intensity, I, is a measure of the amount of energy per time that passes through a given area. Since energy per time is power, P, the intensity of a wave is

$$I = \frac{P}{A} \tag{14-5}$$

If a source of sound is small – or if we, as the observers, are far away from it – it acts like a point source. The sound that emanates from it, then, moves out equally in all directions (on "spherical wavefronts"). Remembering the surface area of a sphere of radius r is

$$A_{sphere} = 4\pi r^2$$

we can calculate the intensity of a point source from Equation 14-5:

Point Source

If a point source emits sound with a power, P, and there are no reflections, the intensity at a distance r from the source is

$$I = \frac{P}{4\pi r^2} \tag{14-7}$$

Why do we need to qualify the above with the phrase "if there are no reflections"?

Do a thought experiment: if we put a point source next to a wall, half of the sound comes directly toward us and half hits the wall. The half of the sound that hits the wall reflects, and moves toward us slightly later. In effect, the intensity of the source doubles.

Note that doubling the intensity of a sound *does not* double its "loudness":

Human Perception of Loudness

The intensity of a sound must be increased by a factor of 10 in order for it to seem twice as loud to our ears.

[Do Example 14-3 on p 465.]

Intensity Level and Decibels

The intensity level, β, of a sound gives an indication of how loud it sounds to our ears. The intensity level is defined as follows:

$$\beta = 10 \log (I/I_0) \tag{14-8}$$

The value of β is given in decibels.

[Do Example 14-4 on p 467.]

14.6 The Doppler Effect

Let's do another thought experiment. Imagine you're standing on the side of the road as a car passes by, and the driver of the car is pressing on his horn. What do you hear?

Of course, the pitch is higher as the car is approaching you, and lower as it's moving away from you. This is a property of waves in general: the change in frequency due to relative motion between a source and a receiver is called the Doppler effect.

Moving Observer

Suppose an observer is moving with a speed u relative to a stationary source. If the frequency of the source is f, and the speed of the waves is v, the frequency f' detected by the observer is

$$f' = (1 \pm u/v) f \tag{14-9}$$

The plus sign applies to the observer approaching the source, and the minus sign to the observer receding from the source. Where does this come from?

An observer moving toward the source effectively increases the speed of the waves, without affecting the wavelength. If the sound waves emitted from the source at rest have a frequency f, a wavelength λ, and a speed $v = \lambda f$, then the speed (letting $v' = v \pm u$) will change the frequency:

$$f = \frac{v}{\lambda} \quad \text{so} \quad f' = \frac{v \pm u}{\lambda} = \frac{v \pm u}{v/f} = \left(\frac{v \pm u}{v}\right) f = \left(1 \pm \frac{u}{v}\right) f$$

...while leaving the wavelength unaffected (so we can still substitute $\lambda = v/f$).

A moving source changes the frequency and wavelength of a wave (see Figure 4-16 on page 470), but doesn't affect its velocity. In that case, we write

$$v = \lambda f \quad \text{so} \quad v' = \lambda' f'$$

But how do we find λ' or f'? (Check out the third paragraph of p 470.) **[I strongly encourage you to read through this derivation and home to understand this. Do Example 14-5 on p 469.]**

Moving Source

If the source is moving with a speed u and the observer is at rest, the observed frequency is

$$f' = \left(\frac{1}{1 \mp u/v}\right) f \tag{14-10}$$

The minus sign applies to the source approaching the observer, and the plus sign to the source receding from the observer. We can combine both of these results and arrive at the general case:

General Case

If the observer moves with a speed u_o and the source moves with a speed u_s, the Doppler effect gives

$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right) f \tag{14-11}$$

The meaning of the plus and minus signs is the same as for the moving-observer and the moving-source cases given above. When people refer to "The Doppler Effect", they're usually referring to this specific formula.

[Do Example 14-6 on p 471.]

14.7 Superposition and Interference

Waves can combine to give a variety of effects.

[Open to p 474 and explain these effects:]

Superposition

When two or more waves occupy the asme location at the same time, they simply add, $y_{\text{total}} = y_1 + y_2$.

Constructive Interference

Waves that add to give larger amplitude exhibit constructive interference.

Destructive Interference

Waves that add to give smaller amplitude exhibit destructive interference.

Interference Patterns

Waves that overlap can create patterns of constructive and destructive interference. These are referred to as interference patterns.

In Phase/Opposite Phase

Two sources are in phase if they both emit crests at the same time. Sources have opposite phase if one emits a crest at the same time the other emits a trough.

[Do Example 14-7 on p 476.]

14.8 Standing Waves

[These aren't strictly necessary concepts. So run through them quickly if there's extra time during class.] Take a look back at Figures 14-7 and 14-8 on p 458. Recall that a wave reflects when it reaches the end of the medium through which it travels. (A wave on a string reflects off the wall or pole to which it's stuck.) What if we shrunk the length of that string down, smaller and smaller and smaller until it was only the width of that pulse?

The string would be constantly bouncing back and forth from the two ends – it would be "fixed" in one spot. This is called a standing wave. Standing waves oscillate in a fixed location.

Waves on a String

The fundamental, or first harmonic, corresponds to half a wavelength fitting into the length of a string. The fundamental for waves of speed v on a string of length L is

$$f_1 = \frac{v}{2L} \tag{14-12}$$

$$\lambda_1 = 2L$$

The higher harmonics, with $n = 1, 2, 3, ...$, are described by

$$f_n = nf_1 = n(v/2L) \tag{14-13}$$

$$\lambda_n = \lambda_1/n = 2L/n$$

Note that the length of the string L *must* be some multiple of $\lambda_n/2$, since

$$\lambda_n = 2L/n \rightarrow L = n(\lambda_n/2)$$

Can you explain why this has to be the case? **[If it were any other value, the waves on the string would destructively interfere. Do Example 14-8 on p 480.]**

Vibrating Columns of Air

[This section can probably be skipped over.]

The harmonics for a column of air closed at one end are

$$f_n = nf_1 = n(v/4L) \quad n = 1, 3, 5, ... \tag{14-14}$$

$$\lambda_n = \lambda_1/n = 4L/n$$

The harmonics for a column of air open at both ends are

$$f_n = nf_1 = n(v/2L) \quad n = 1, 2, 3, ... \tag{14-15}$$

$$\lambda_n = \lambda_1/n = 2L/n$$

In both of these expressions the speed of sound is v and the length of the column is L.

14.9 Beats

Beats occur when waves of slightly different frequencies interfere.

They can be thought of as interference patterns in time. To the ear, beats are perceived as an alternating loudness and softness to the sound. Follow along with the derivation on pages 485-486 to see how we arrive at the following equation:

Beat Frequency

If waves of frequencies f_1 and f_2 interfere, the beat frequency is

$$f_{\text{beat}} = |f_1 - f_2| \tag{14-18}$$

Beats are explained visually in Figure 14-30 on p 486.

[Do Example 14-10 on p 487.]