Lecture Notes: Chapter 28

Physical Optics: Interference and Diffraction

Notes by A. W. Watson for La Salle University, Spring 2016.
[This is a 3-day lecture.]
Outline taken from pages 1000-1002 of Walker's Physics, 4th. Ed. (2014).

Last chapter, we discussed some of the basics of wave motion: wavelength, frequency, reflections, and superposition. But waves can be very complex creatures, exhibiting all sorts of phenomena which affect things like their intensity, direction, and our ability to see them. In this chapter, we investigate some of these more complex properties of wave motion.

28.1 Superposition and Interference

Whether you're aware of it or not, you experience waves in superposition every day. Whenever two people are trying to talk at the same time, the sound waves they generate are trying to exist in the same volume of air at the same time – they're "superposed". Because these waves are often very complex and irregular, they're difficult to describe. But if we focus instead on **monochromatic** (or, *single-frequency*) waves, superposition can be imagined graphically as seen in Figure 28-1 on page 977.

The simple addition of two or more waves to give a resultant wave is referred to as superposition. When waves are superposed, the result may be a wave of greater amplitude (constructive interference) or of reduced amplitude (destructive interference).
[See Figure 28-2 on page 977.]

Monochromatic Light

Monochromatic light consists of waves with a single frequency and, hence, a single color (or, as is the case with sound, a single pitch).

Recall last chapter when we discussed "phase shifts": these changes in "starting position" of a wave are especially important when looking at interference phenomena, like wave superposition. **[What is a phase shift?]** Monochromatic waves which have the same phase shift are said to be *in phase*, or *coherent*:

Coherent / Incoherent Light

Light waves that maintain a constant phase relationship with one another are referred to as coherent. Light waves in which the relative phases vary randomly with time are said to be incoherent.

Where have you encountered coherent light before? What about incoherent light?

Lasers emit coherent monochromatic light, while light bulbs emit incoherent white light.

Can two rays of light with different frequencies be coherent? Why or why not?

They cannot be coherent because it's impossible for them to maintain a constant phase relationship with one another.

But when do waves interfere constructively and when do they interfere destructively? To interfere perfectly constructively, they must have a phase difference of an integer number of wavelengths $(..., -2\lambda, -\lambda, 0, \lambda, 2\lambda, ...)$ and to interfere perfectly destructively, they must have a phase difference of an integer number of wavelengths *minus one half of one wavelength* $(..., -\frac{5}{2}\lambda, -\frac{3}{2}\lambda, -\frac{1}{2}\lambda, \frac{1}{2}\lambda, \frac{3}{2}\lambda, ...)$. **[Why? Draw on board.]**

If we send a monochromatic wave from two sources at once, and they're initially in phase, then whether or not they're in phase when they arrive at an observer depends on how far the observer is from each of the sources. **[Draw this on the board.]** **Example 28-1 on page 978** explains this concept well.

[Exercise 4 on page 1003, Exercise 6 on page 1004.]

28.2 Young's Two-Slit Experiment

Isaac Newton is known for his work in several fields of science.

Can anyone name some of them?

Three Laws of Motion, Law of Universal Gravitation, invention of Calculus, experiments with lenses, etc.

His work in the field of optics relies on the idea that light is made up of tiny particles (which he called "corpuscles"), building on the atomic theory of matter of the ancient Greeks. But light can also be modeled as a wave, using similar equations to those used for sound. So is light really a particle, or is it a wave? In 1801, English physician and physicist Thomas Young created an experiment which he interpreted as supporting the idea of "light as a wave".

Interference effects in light are shown clearly in Young's two-slit experiment, in which light passing through two slits forms bright and dark interference "fringes" (this, of course, is a result of **Huygen's principle**, which says that "if light is a wave... each slit acts as the source of new waves, analogous to water waves passing through two small openings" (979)). Since it's impossible for particles to exhibit interference patterns, Young believed that this proved light was a wave.

Does anyone know the problem with Young's interpretation?

It's run "too quickly". If he could stop it after only a few milliseconds, he could see the interference pattern building up photon-by-photon.

But what does this mean?

Light exhibits both wave-like and particle-like properties. Hence *wave-particle duality*.

Take a look back at pages 476 and 477 (specifically Example 14-7 and Active Example 14-2). Here, we had a person standing a certain distance from two speakers which emitted a sine wave in the same or opposite phase, but this is a very specific case. As always, it's easiest to begin with a much more general case, and then work our way toward specifics. So let's take a look at Figure 28-5 on page 980.

Since the slit separation is generally much smaller than the distance to the "distant screen" (see Figure 28-3 on page 979), we can say that the rays of light arrive more or less parallel, and so the approximate path difference is just

$$\Delta\ell = d\sin\theta$$

Note that, with a two-slit experiment, there is always a central bright fringe on the distant screen, because this point is the exact same distance from each slit. Here, the two rays of light are in phase with each other and interact constructively. In general, if two parallel rays start in phase, and end in phase, the path difference can *only* be an integer number of wavelengths, as seen in Figure 28-5 on page 980.

$$\Delta\ell = m\lambda = d\sin\theta \quad \text{where } m \text{ is an integer} \quad (m \in \mathbb{Z})$$

Conditions for Bright Fringes

Bright fringes in a two-slit experiment occur at angles θ given by the following relation:

$$d\sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, ... \tag{28-1}$$

In this expression, λ is the wavelength of the light and d is the separation of the slits. The various values of the integer m correspond to different bright fringes.

Conditions for Dark Fringes

The locations of dark fringes in a two-slit experiment are given by the following:

$$d\sin\theta = \left(m - \frac{1}{2}\right)\lambda \qquad m = 1, 2, 3, ... \quad \text{(above central bright fringe)} \tag{28-2}$$

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad m = -1, -2, -3, ... \quad \text{(below central bright fringe)}$$

Or, more succinctly:

$$d \sin\theta = \pm\left(m - \frac{1}{2}\right)\lambda \qquad m = 1, 2, 3, ...$$

And then, fairly obviously, as can be seen in Figure 28-7 on page 981:

Linear Distance

If the screen on which the interference pattern is projected in a two-slit experiment is a distance L from the slits, the linear distance to a given bright or dark fringe is $y = L\tan\theta$.

[Exercises 12, 16, 18 on page 1004.]

28.3 Interference in Reflected Waves

I got a new phone recently, and bought a screen protector with it at the store. The store employee took it upon himself to open the screen protector and put it on my phone without asking if I'd like him to do it, and not *only* did he get a little bit of string stuck underneath it, he also didn't apply it correctly and now there's some air stuck underneath it.

Most people try to be very careful when applying things like screen protectors to their tablets and phones because even a little bit of air stuck underneath can create "rainbow" effects in the trapped air bubbles. This small bit of trapped air means that the light reflected from the screen protector and the light reflected from the phone beneath the screen protector travel slightly different paths and can interfere. Light waves reflected from different locations can interfere, just like light from the slits in a two-slit experiment.

Recall that a string with a fixed end (tied to a wall or a post) inverts (goes through a $\pi = 180°$ phase shift) upon reflection, and a string with an end that's free to move laterally does not. Light waves do something similar.

Every material has an **index of refraction**, $n = c/v$, where c is the speed of light in a vacuum, and v is the speed of light in that material (we'll discuss this a bit more in Chapter 26). The index of refraction of a vacuum is exactly 1; air is about 1.0003; water about 1.33; glass about 1.50. The higher a material's index of refraction, the more "optically dense" it is. Just like a wave on a string inverts when reflecting off of a "dense" wall, a light wave inverts when reflecting off an optically dense material (denser than the original material).

[Practice: calculate the speed of light through air; through water.]

Phase Changes Due to Reflection

No phase change occurs when light is reflected from a region with a lower index of refraction, whereas a $180°$ (half-wavelength) phase change occurs when light reflects from a region with a higher index of refraction, or from a solid surface.

With that in mind, we can investigate several other instances of interference patterns caused by reflection. We can start with the **air wedge**, illustrated in Figure 28-8 on page 984. "Ray 1" doesn't undergo a phase change when it reflects, because it's travelling through glass (high n) and reflects off of air (low n), while "Ray 2" travels approximately $2d$ (the plate separation is actually very, very small), and also undergoes a $180°$ phase change, because it travels through air and reflects off of glass.

For the two Rays to interfere constructively, then, the difference in path lengths, plus the $180°$ phase shift of Ray 2, must equal an integer number of wavelengths. To put it mathematically:

$$2d + 180° = 2d + \frac{\lambda}{2} = m\lambda \qquad \text{or} \qquad m = \frac{2d}{\lambda} + \frac{1}{2}$$

[Exercise 28 on page 1005.]

Air Wedge

Two plates of glass that touch on one end and have a small separation on the other end form an air wedge. When light of wavelength λ shines on an air wedge, bright fringes occur when the separation between the plates, d, is such that

$$\frac{1}{2} + \frac{2d}{\lambda} = m \qquad m = 1, 2, 3, ... \tag{28-5}$$

Similarly, dark dringes occur when the following conditions are satisfied:

$$\frac{1}{2} + \frac{2d}{\lambda} = m + \frac{1}{2} \qquad m = 0, 1, 2, ... \tag{28-6}$$

Or, of course, when

$$m = \frac{2d}{\lambda} \qquad m = 0, 1, 2, ...$$

Newton's Rings

When a piece of glass with a spherical cross section is placed on a flat sheet of glass, the resulting interference fringes form a set of concentric circles known as Newton's rings. The geometry here is a bit more complex, but in general, for a curved surface in contact with a flat surface, the radius of the N^{th} Newton's ring (bright) is given by

$$r_N = \left[\left(N - \frac{1}{2}\right)\lambda R\right]^{1/2}$$

where R is the radius of curvature of the curved piece of glass (for a spherical lens, $R =$ the radius of the sphere). **[+5% bonus to anyone who can get me a GOOD derivation (with explanations of each step) of this before the next exam.[1]]**

[1] Equation of circle: $(x-h)^2 + (y-k)^2 = r^2$. Center spherical lens of radius R at $(0, R)$, and look at point $(x, y) = (r, d)$, where $d \ll R$. Then $r^2 + (d-R)^2 = R^2$. Solve for $d = R - \sqrt{R^2 - r^2}$, and use binomial approximation to arrive at $d \approx (1/2)(r^2/R)$. Substitute d into air wedge equation, letting $m = N$, and solve for r.

Thin Films

Thin films, like those in a soap bubble, can produce colors in reflected light by eliminating other colors with destructive interference. Thin film interference is what gives bubbles their rainbow coloration.

Thin films are handled only slightly differently from the way we looked at air wedges and Newton's rings. The process is outlined on pages 986 and 987, but essentially, what we do is look at effective path lengths.

Look at Figure 28-12 on page 986. Ray 1 hits the thin film and splits in two: between its first contact with the film and its last contact (which, for Ray 1 are the same point), Ray 1 has undergone a phase change, but has not travelled any physical distance. So, effectively, we can think of Ray 1 as having moved through a distance of $\lambda/2$, to account for the phase change.

Ray 2 has travelled back and forth through the film, moving a physical distance of $2t$, but has not undergone any phase change. Therefore, effectively as well as actually, Ray 2 has travelled a distance of $2t$ between its first and last points of contact with the thin film. For constructive interference, the difference in effective paths should be an integer multiple of the wavelength, or

$$2t - \frac{\lambda}{2} = m\lambda \qquad m = 0, 1, 2, ...$$

To put this in a useful form, though, we need to remember that the index of refraction of a material gives the ratio between the speed of light in a vacuum and the speed of light in that material, $n = c/v$. It just so happens (as we'll learn in Chapter 26) that light rays travelling through materials with different indices of refraction change wavelength, and n gives the ratio between those different wavelengths: $n = \lambda_{vacuum}/\lambda$. This means that we can write

$$\frac{2t}{\lambda} - \frac{1}{2} = m \qquad \text{or} \qquad m = \frac{2nt}{\lambda_{vacuum}} - \frac{1}{2} \qquad m = 0, 1, 2, ...$$

for **constructive thin film interference** and

$$m = \frac{2nt}{\lambda_{vacuum}} \qquad m = 0, 1, 2, ...$$

for **destructive thin film interference** (by just adding one half-wavelength to the effective path difference). Note that the interference pattern depends on both the wavelength of the light, and the thickness of the film.

Knowing this, why do soap bubbles and screen protectors exhibit "rainbow" patterns?

Because white light, which is composed of light of all colors, contains light of many different wavelengths, which interfere constructively and destructively at different points on the bubble / thin film.

[Exercises 32 and 34 on page 1005, 36 and 42 on page 1006.]

28.4 Diffraction

When a wave encounters an obstacle, or passes through an opening, it changes direction. This phenomenon is referred to as diffraction. We've already encountered this when discussing Huygen's principle in Section 28-2. Diffraction (not to be confused with *refraction*) is the bending of a wave around an obstacle. Diffraction is easily seen with water waves, and heard with sound waves. The fact that you can hear someone talking in the next room, even though you don't have a direct line of sight with them, is partly thanks to the diffraction of sound waves, moving around obstacles from the source (the other person) to the receiver (you).

We know that light can self-interfere after passing through a pair of slits, because the two slits act as separate point sources. But light can also self-interfere after passing through just a single slit. While it may seem counterintuitive at first, we can work through the problem a bit and discover how this makes sense.

Single-Slit Diffraction

When monochromatic light of wavelength λ passes through a single slit of width W, it forms a diffraction pattern of alternating bright and dark fringes.

Take a look at Figure 28-19 on page 990. In Figure 28-19 (a), we artificially cut the slit into a "top" and a "bottom" part. For every ray 1, 2, or 3 on the top, there is a corresponding ray 1', 2' or 3' on the bottom. If $\theta = 0$, all of the rays travel straight through, travelling the same distance, and constructively interfering, creating a central bright fringe of width W.

If the rays travel at some angle, $\theta \neq 0$, though, they may interfere constructively or destructively, depending on the path difference between the top rays and the bottom rays. We see the first dark fringe where

$$\Delta\ell = \frac{1}{2}\lambda = \frac{1}{2}W\sin\theta \qquad \text{or} \qquad W\sin\theta = \lambda$$

We perform a similar procedure to find the second dark fringe, the third dark fringe, and so on — artificially breaking the slit into further and further subdivisions. If you follow the procedure on pages 990–991, you'll see that, in general, the positions of the dark fringes satisfy:

Condition for Dark Fringes

The condition that determines the location of dark fringes in single-slit diffraction is

$$W\sin\theta = m\lambda \qquad m = \pm1, \pm2, \pm3, ... \tag{28-12}$$

Bright Fringes

Bright fringes are located approximately halfway between successive dark fringes. In addition, the central bright fringe is approximately twice as wide as the other bright fringes.

Why is this?

Recall the procedure we used to construct the first dark fringe, where a "top" ray and a "bottom" ray interfered. For the first dark fringe "above" the central bright fringe, $\theta > 0$, and for the first dark fringe "below" the central bright fringe, $\theta < 0$, but θ is the same in either case. Similarly, the central bright fringe can be thought of as two interference fringes, at $\pm\theta$, only this time, they happen to be right next to each other. (Also, these fringes can be modeled by a $\operatorname{sinc} x = (\sin x / x)$ curve.)

How does the width of the central bright fringe change as we decrease the width of the slit?

Although it seems counterintuitive, decreasing the width of the slit increases the width of the central bright fringe. The first dark fringe occurs at $\theta = \arcsin(\pm\lambda/W)$. If λ is unchanged, decreasing W means increasing λ/W, which gives $\theta \to \pm\pi$.

(If time.) Read the first paragraph on page 993. Why should there be a bright spot in the middle of this shadow?

Because light diffracting around the edge of the coin has to travel the same distance to reach the center of the shadow, which creates a constructive interference "bright spot".

Question 7 on page 1003.) Why is the central spot in Newton's rings dark?

consider two beams of light: one reflects off the bottom of the circular glass lens and has no phase shift, the other travels a miniscule distance through the air and reflects off of the flat piece of glass underneath, undergoing a $180°$ phase change, with practically no additional *physical* path length added. These two beams are then $180°$ out of phase and cancel.

[Exercises 46 and 50 on page 1006.]

28.5 Resolution

Most people have heard the term "resolution". But primarily with regards to the resolution of computer, television, or smartphone screens. In this sense, resolution is usually measured in "ppi" or pixels per inch. In physics, we have a slightly different interpretation of resolution: it refers to the ability of a visual system, like the eye or a camera, to distinguish closely spaced objects.

For a human being, that limit is an *angular separation*[2] of about $0.0128°$[3]. Given an arm length of 1 m, this means that an average human being is able to resolve the thickness of a thick human hair at arm's length.[4] Of course, telescopes and cameras, and even

[2] **Review this:** angular separation is found by drawing a line from one object, to an observer, and then to a second object. The measure of that angle is the angular separation of the two objects.

[3] https://www.astro.umd.edu/ thuard/astr288c/lecture6-notes.pdf

[4] Also interesting: http://lawnchairanthropology.com/2013/05/11/arm-and-leg-modelling/

many animals, have greater resolution than the human eye.

Resolution is of extreme importance to astronomers trying to observe stars. Stars are so far away and so compact, that they're essentially perfect point sources of light. As you might expect, this means they have circular diffraction patterns which surround them, called **Airy patterns** or Airy disks, named after George Biddell Airy, who first described them mathematically in 1835[5].

Any point source or circular opening through which light is transmitted generates this circular diffraction pattern, which can be seen in Figure 28-21 on page 993. As two point sources get closer and closer together, as seen by the observer (as their *angular separation* decreases), the patterns begin to overlap, as can be seen in Figure 28-22 on page 994. Mathematically, we find that...

First Dark Fringe

A circular aperture of diameter D produces a circular diffraction pattern in which the first dark fringe occurs at the angle θ given by the following condition:

$$\sin\theta = 1.22\frac{\lambda}{D} \tag{28-14}$$

This can be described qualitatively and quantitatively by Rayleigh's criterion:

Rayleigh's Criterion: Qualitative Statement

Rayleigh's criterion states that two objects become blurred together when the first dark fringe of one object's diffraction pattern passes through the center of the other object's diffraction pattern.

Rayleigh's Criterion: Quantitative Statement

In quantitative terms, Rayleigh's criterion states that if the angular separation between two objects is less than a certain minimum, $\theta_{\text{min}} = 1.22\lambda/D$, they will appear to be a single object.

A telescope which is limited in resolution only by the Rayleigh criterion, we call *diffraction limited*. Of course, such a telescope would need to be manufactured very precisely, be very clean, and be able to overcome things like atmospheric distortion (if it's a ground-based telescope).

[Exercises 98-101 on page 1009.]

28.6 Diffraction Gratings

Finally, after looking at single- and double-slit diffraction, it seems reasonable to ask what would occur if we had many closely-spaced slits all on one screen. A diffraction grating is a large number of slits through which a beam of light can pass.

[5] "On the Diffraction of an Object-glass with Circular Aperture"

Principal Maxima

The principal maxima produced by a diffraction grating occur at the angles given by the following conditions:

$$d \sin\theta = m\lambda \quad m = 0, \pm 1, \pm 2, ... \tag{28-16}$$

In this expression, d is the distance between successive slits and λ is the wavelength of light. Note that if $m\lambda$ is held constant, then decreasing d necessarily increases $\sin\theta$ — in other words, decreasing the slit separation increases the spread between successive bright peaks in the diffraction pattern.

Number of Lines per Centimeter

Diffraction gratings are often characterized by the number of lines, or slits, they have per centimeter. If the number of lines per centimeter is N, the spacing between slits is $d = 1/N$, where d is measured in centimeters.

Reflection Gratings

Diffraction gratings can also be constructed from a reflecting surface with a large number of reflecting lines, like a CD or a butterfly wing.

Iridescence

When white light shines on a reflection grating, different colors in the light are reflected at different angles. The color effects produced in this way are referred to as iridescence. This can be seen in the photographs on page 1000.

[Review Waves and Particles on pages 1010–1011.]