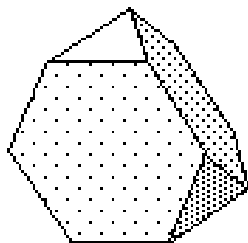




ON THE UNFOLDING OF PRISMS

HONORS THESIS DEFENSE - ANDREW WATSON, '12



Truncated Tetrahedron

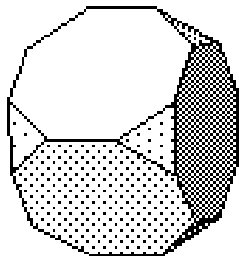
The 13 Archimedean Solids

These all have 2 or more types of regular polygons (e.g. triangles & squares).

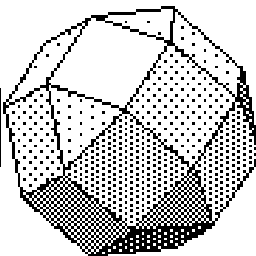
The truncated tetrahedron shows the "progression" from a tetrahedron to another tetrahedron, since the tetrahedron is a dual to itself, i.e., connecting the midpoints of the faces yields another tetrahedron pointing in the opposite direction from the original.

The row below shows the progression from a hexahedron (cube) to an octahedron.

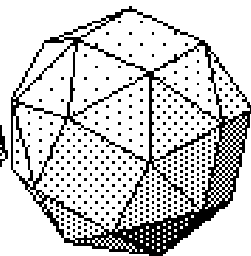
The bottom row shows the progression from a dodecahedron to an icosahedron, as corners are trimmed off and turned into other regular polygons.



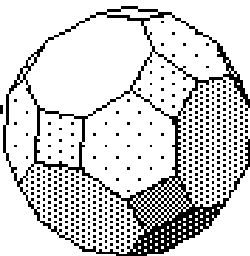
Truncated Cube



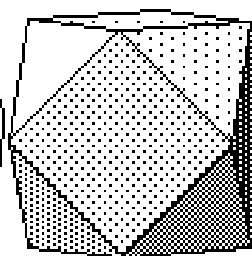
Rhombicub-octahedron



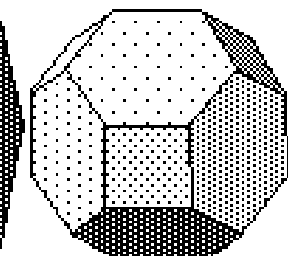
Snub Cube



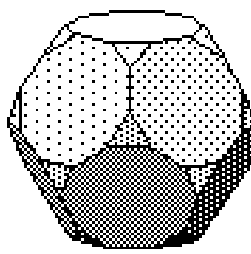
(Rhombi)truncated Cuboctahedron



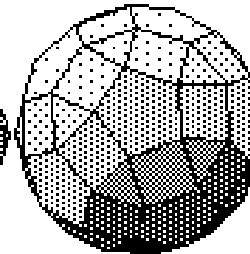
Cuboctahedron (Dymaxion)



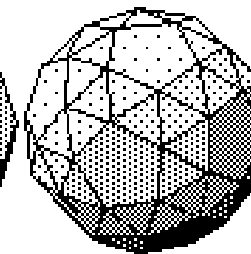
Truncated Octahedron (Meccor)



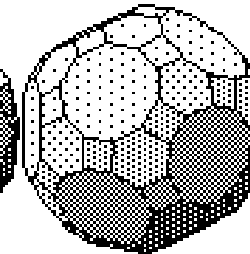
Truncated Dodecahedron



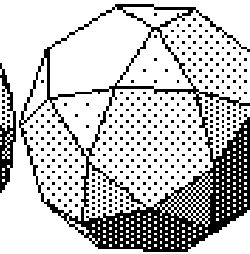
Rhombicosi-dodecahedron



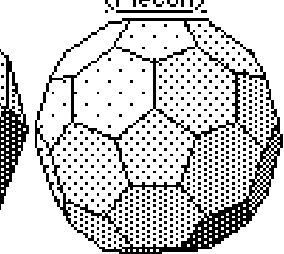
Snub Dodecahedron



(Rhombi)truncated Icosidodecahedron



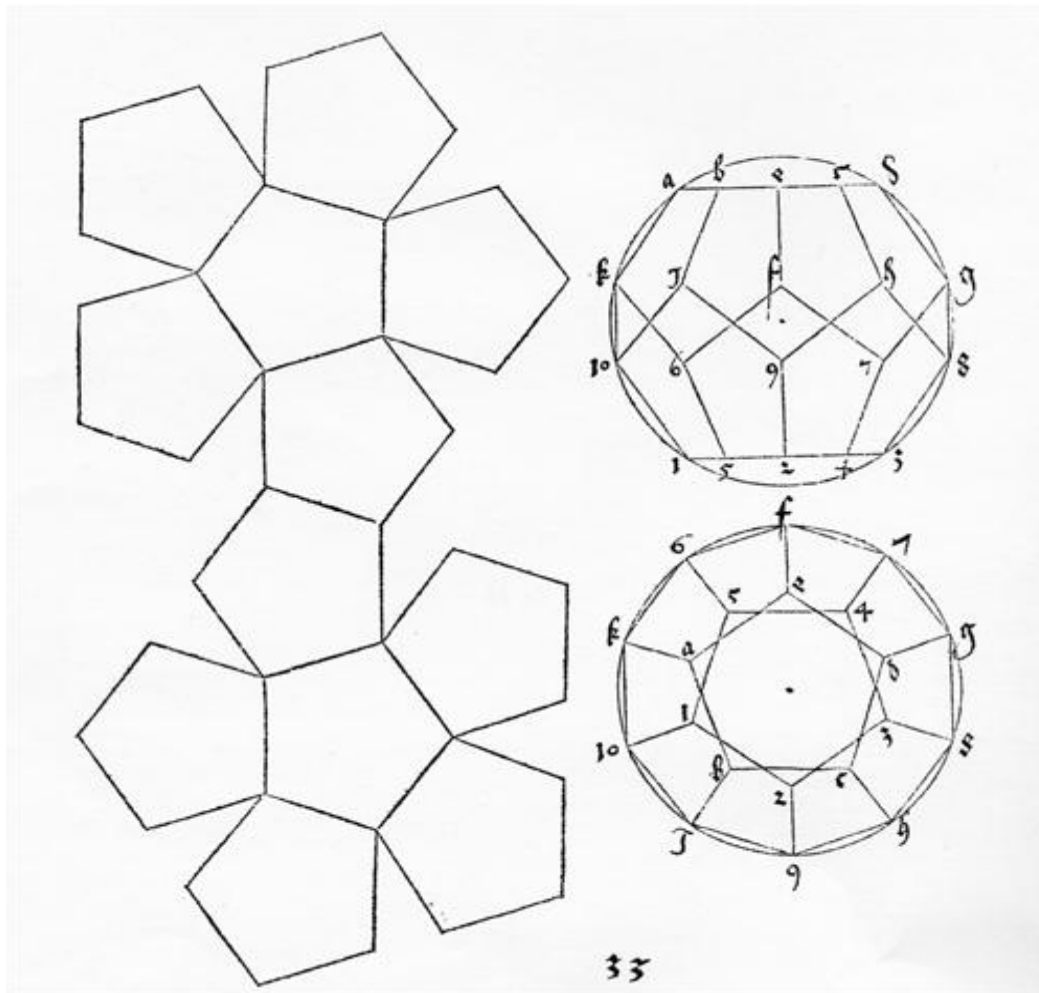
Icosidodecahedron



Truncated Icosahedron

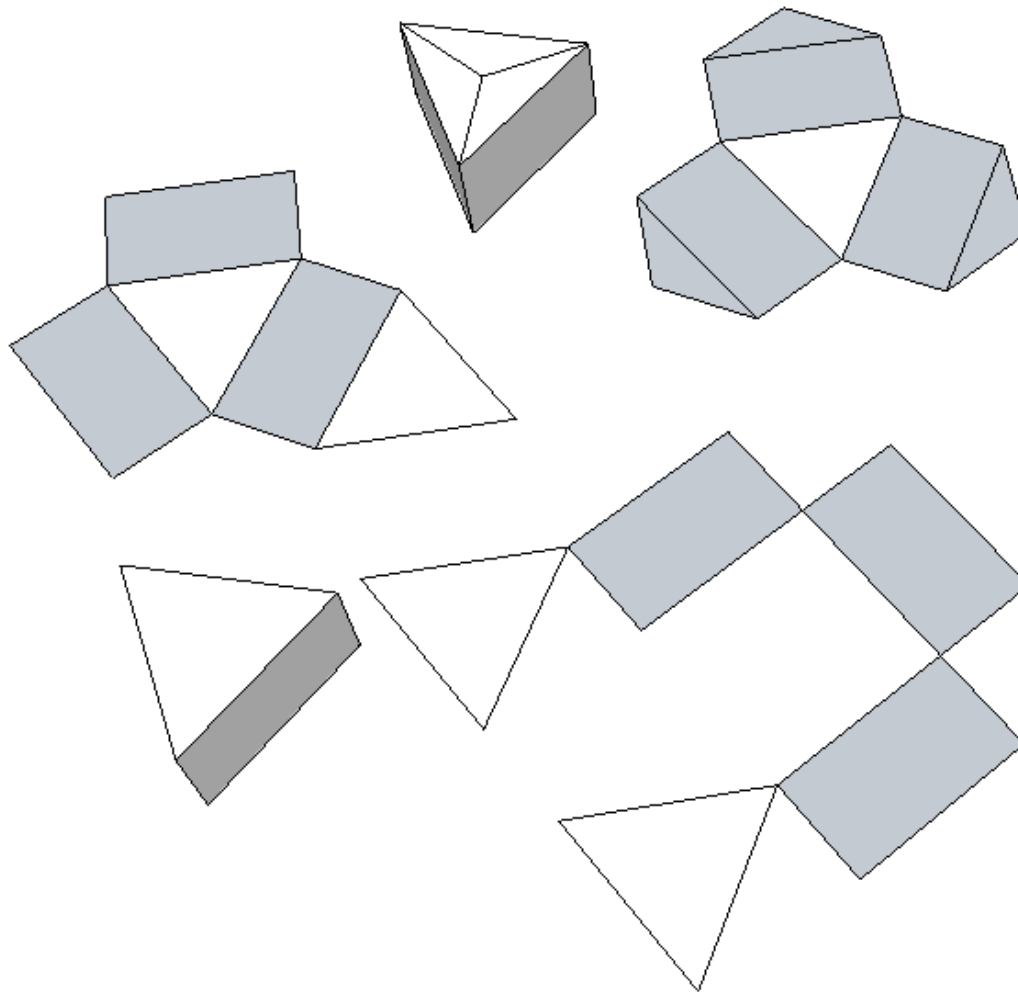
<http://bit.ly/HoVeUb>

Polygons & Polyhedra



Polyhedral “nets”:

- i. Non-overlapping
- ii. Single polygon
- iii. Planar
- iv. Cut only along edges of polygons



Different “types” of unfoldability:
Edge, Vertex, General

Types of Unfoldings for Various Polyhedra

Edge

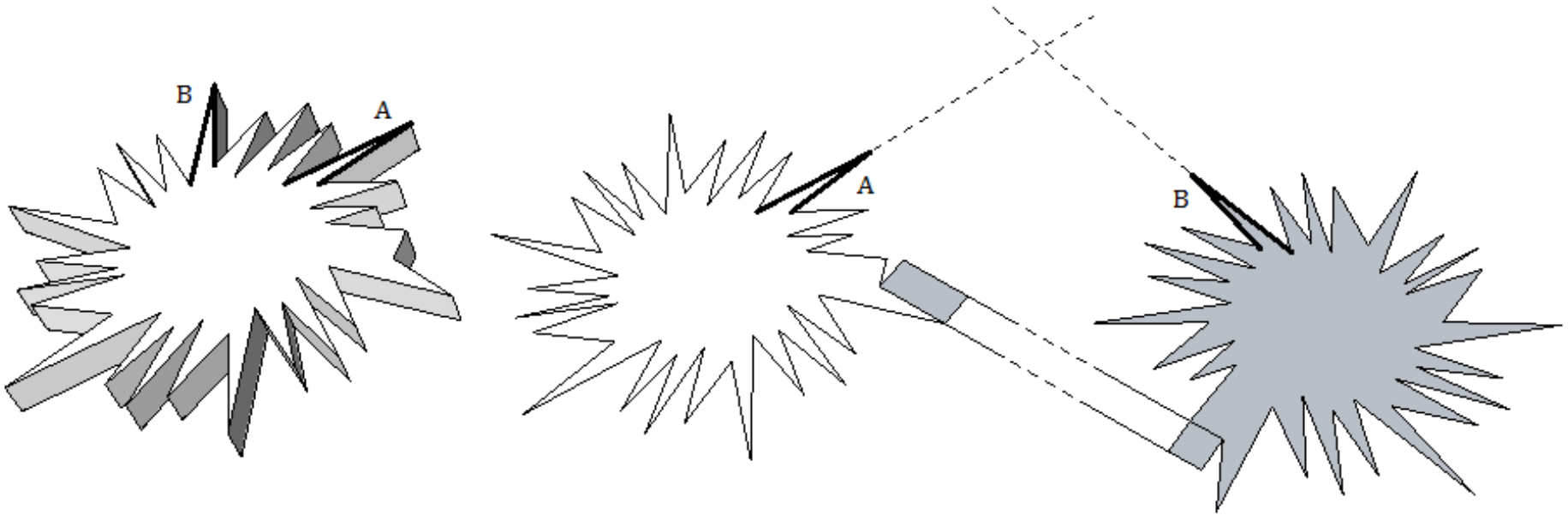
Vertex

General

Types of Unfoldings for Various Polyhedra				Edge	Vertex	General	
Triangulated					YES	YES	
Non-Triangulated	Non-Convex	Prismatoids	Prismoids, etc.	Smooth			
			Prisms, Oblique Prisms, Pyramids		NO		YES?
			Prismoids, etc.	Polygonal	NO		YES?
		Orthogonal Polyhedra			NO	NO	YES
		Convex	Orthogonal Polyhedra	Orthogonal Terrains	Rectangular Prisms	YES	YES
	YES				YES	YES	
	Prismatoids		Prismoids, etc.	Polygonal			YES
					Prisms, Oblique Prisms, Pyramids	YES	YES
	Prismoids, etc.	Smooth	YES	YES	YES		
			YES	YES	YES		
							YES

Theorem 1:

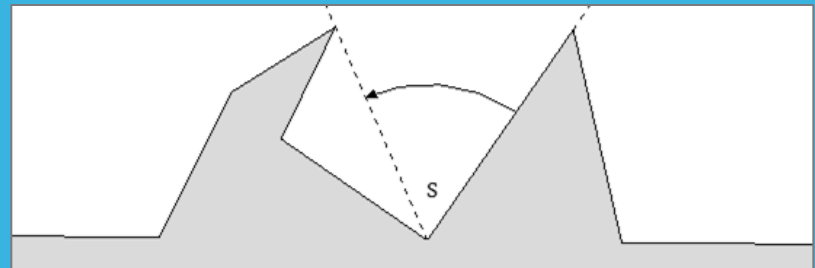
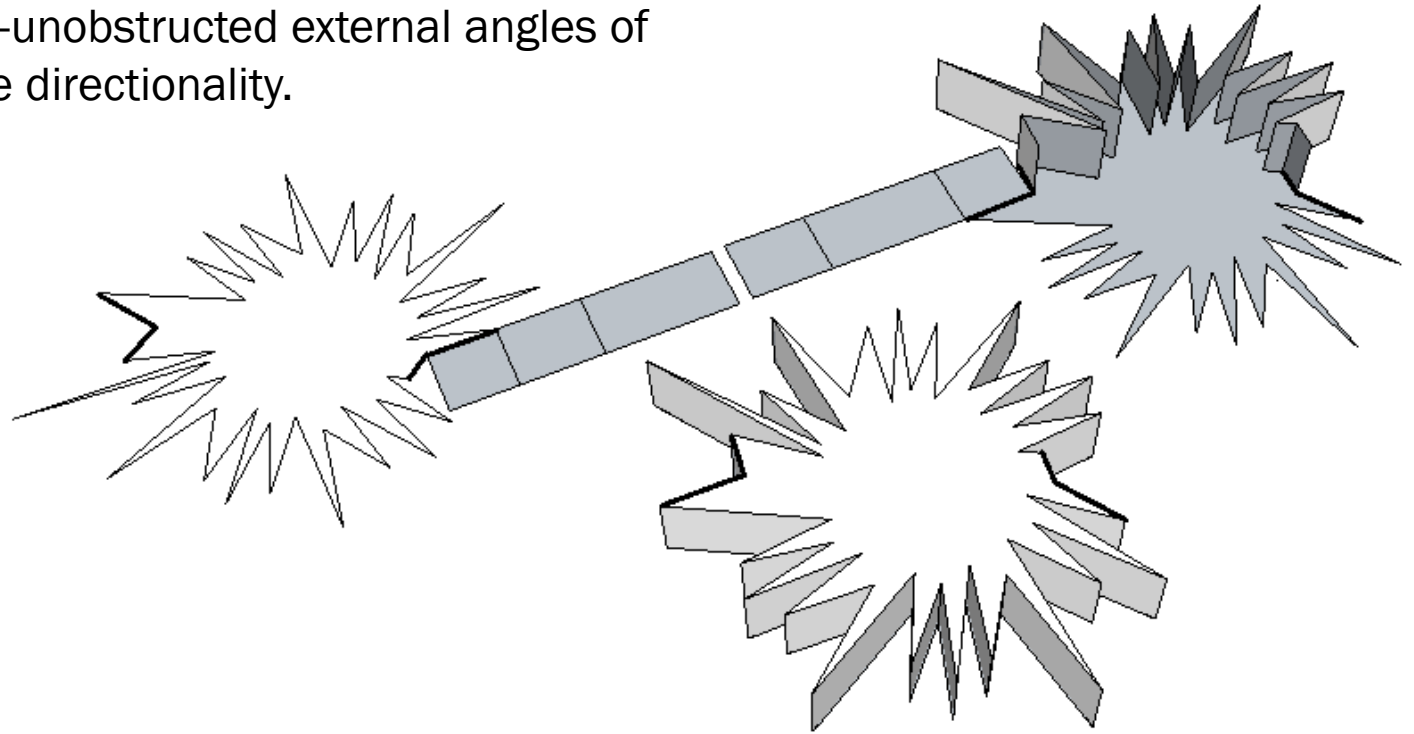
Any genus-0 right prism is unfoldable, given that the base polygon has at least one unobstructed external angle of $\geq 90^\circ$.

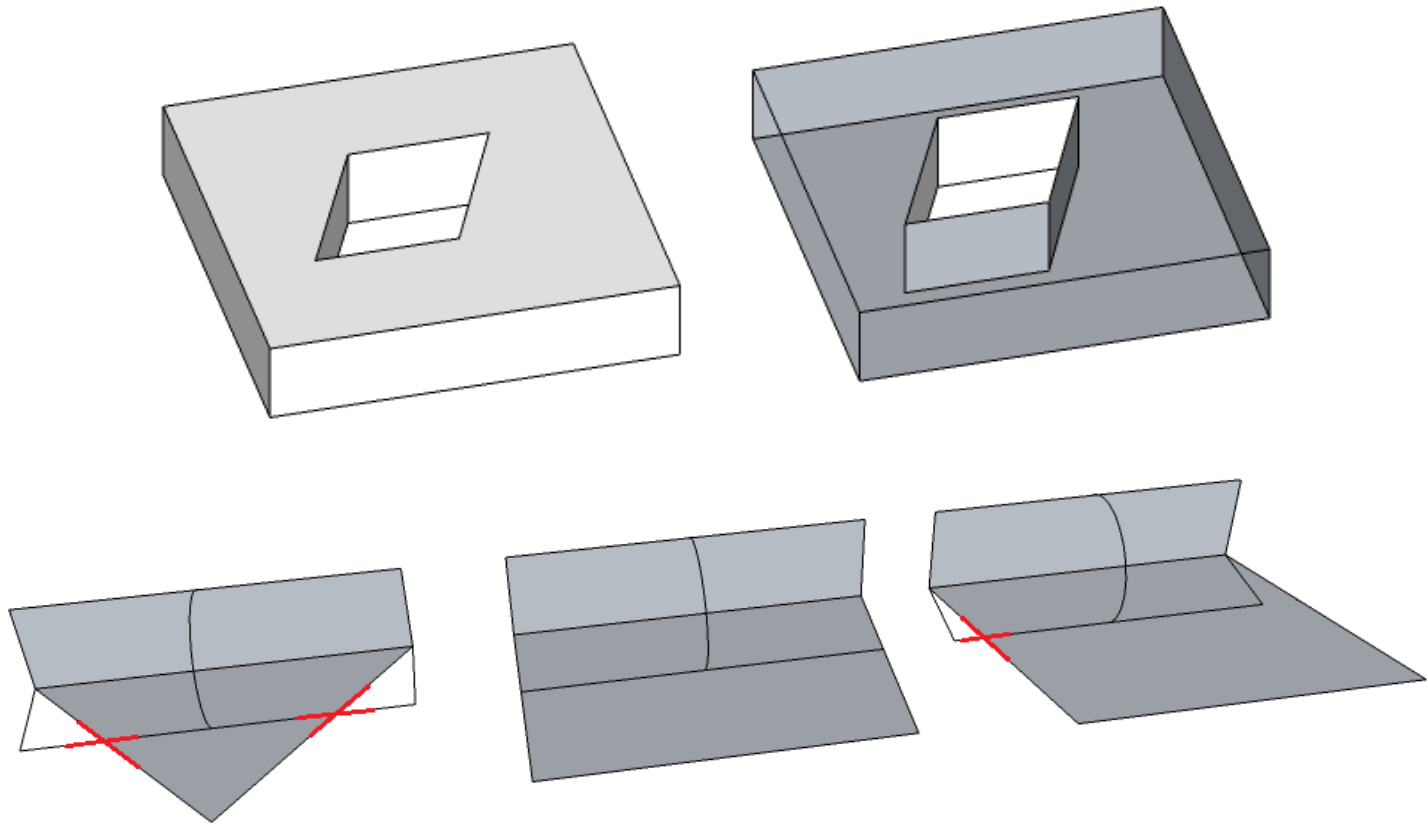


Note: Any spikes A & B, will not crash due to perimeters

Corollary 1-1:

A sufficiently short prism is partially unfoldable if its base has two semi-unobstructed external angles of $\geq 90^\circ$ with opposite directionality.

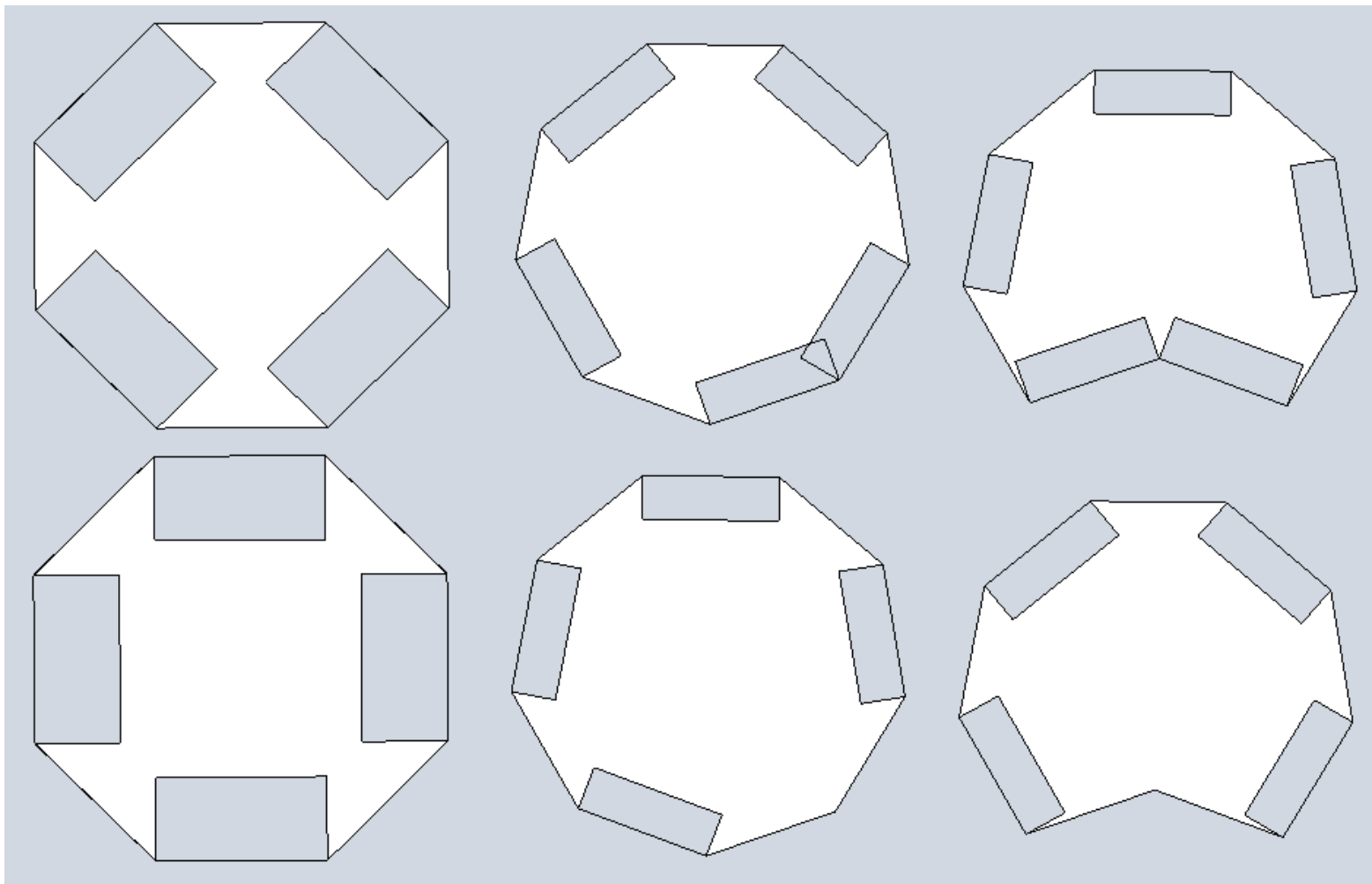




Genus, and how we apply it to polygons

Important characteristics of holes:

- i. Angles of holes
- ii. Number of sides of holes



Theorem 2:

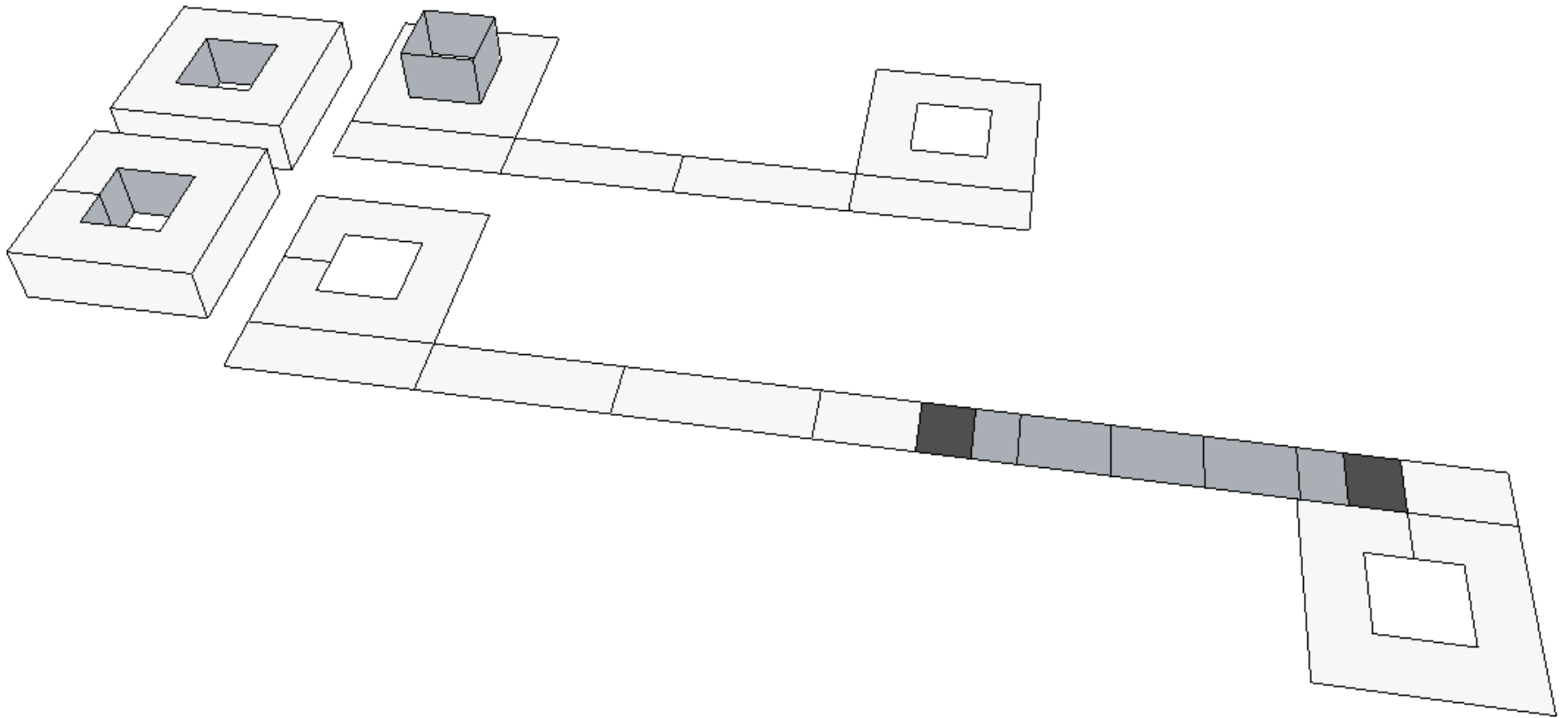
Let P be a properly unfoldable genus-1+ prism of height ϵ . P is edge-unfoldable for sufficiently small ϵ if and only if:

1. All exterior angles of all holes are $\geq 90^\circ$
2. All even-edged holes have ≥ 4 edges
3. All odd-edged holes have ≥ 5 edges
4. All odd-edged holes have at least one exterior angle of $> 180^\circ$

...but what if a prism isn't unfoldable?

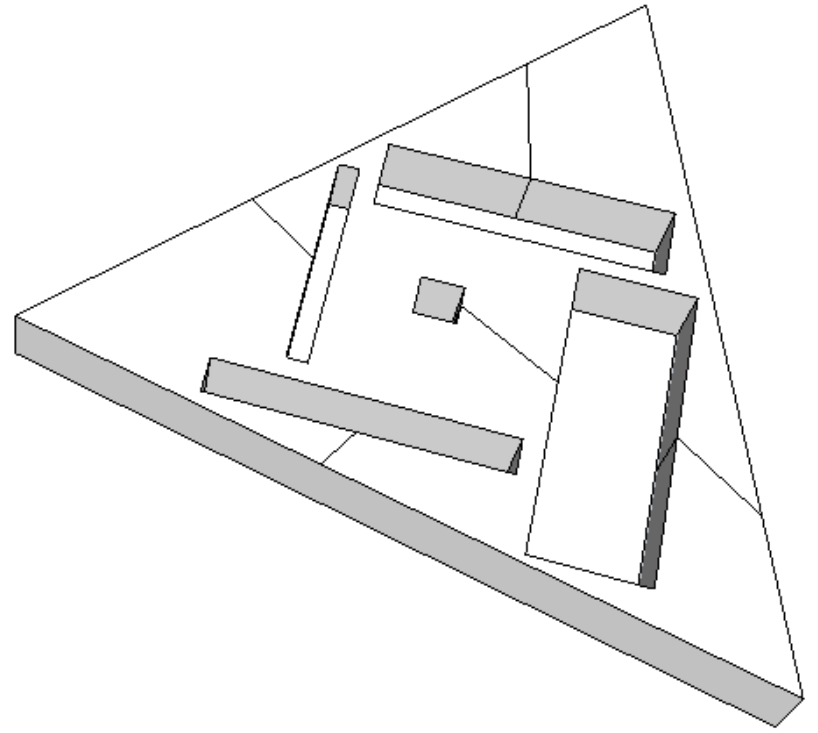
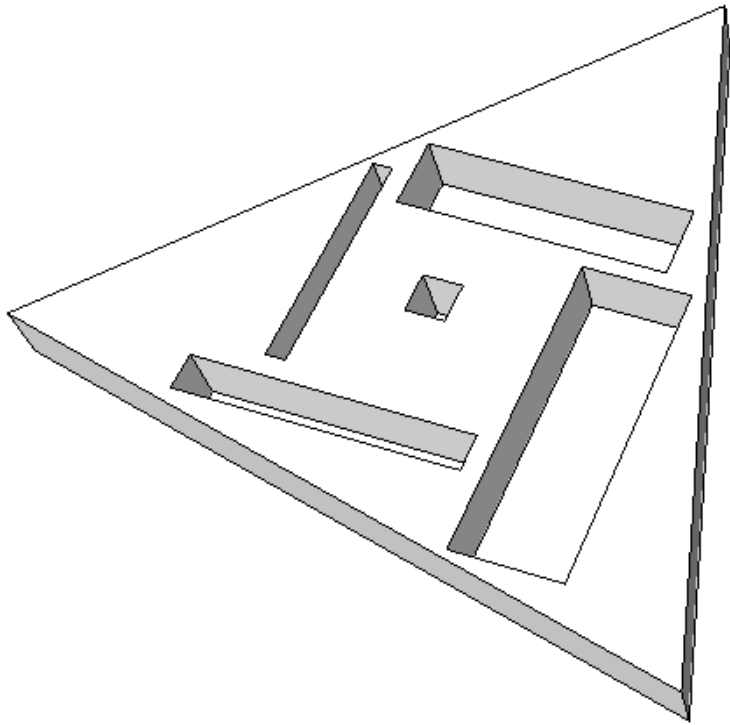
Can we "cut" it (add edges) to make it unfoldable?

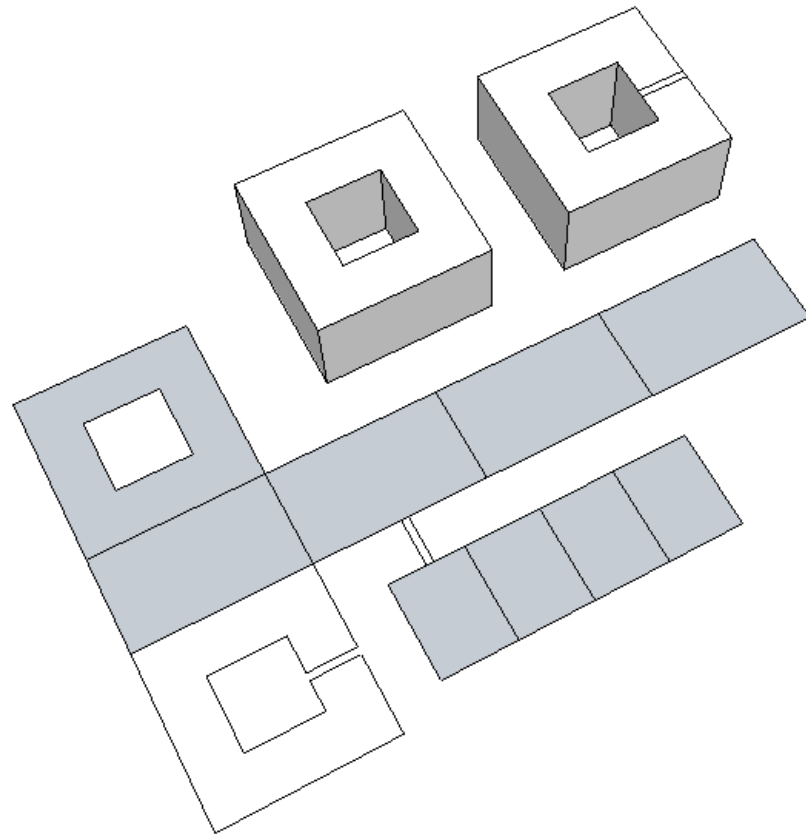




...making a prism unfoldable by adding faces



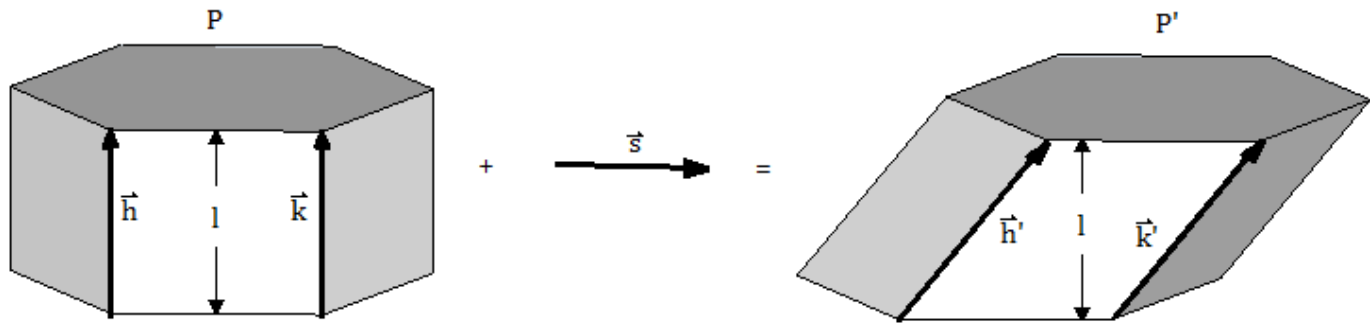




...making a prism unfoldable by adding edges

Relationships between faces are preserved, no matter where the edges are cut...

...therefore, there is a finite number of unfoldings!

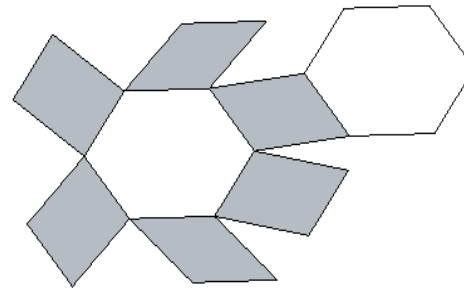
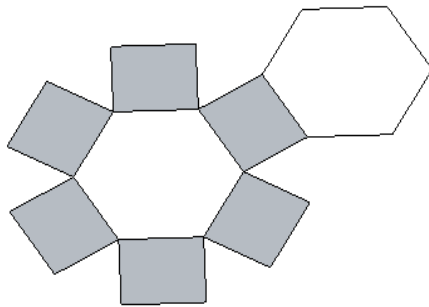


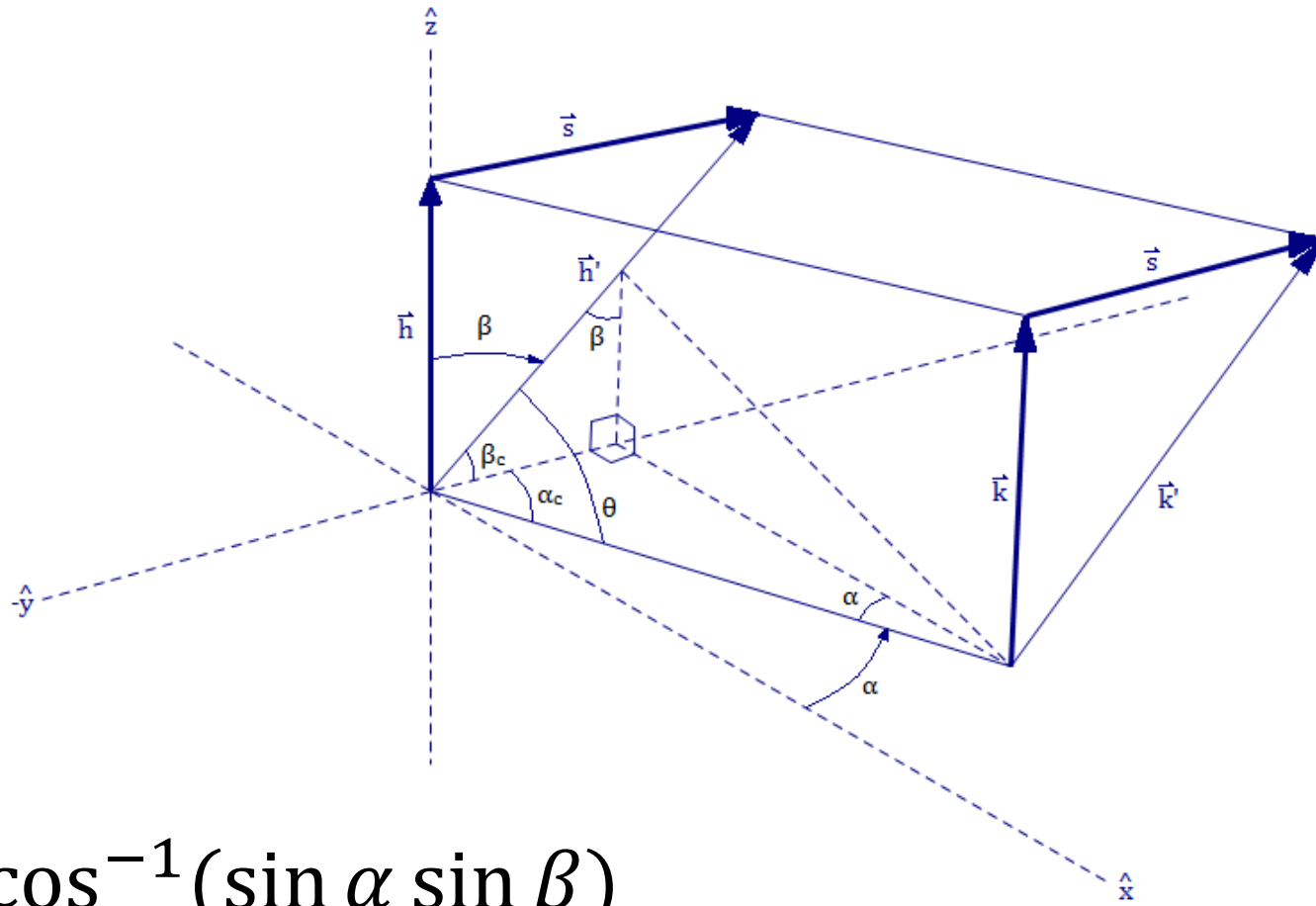
$$\vec{h} \cdot \vec{k} = l^2$$

$$\vec{h}' \cdot \vec{k}' = l^2 + |\vec{s}|^2$$

$$\vec{h} \times \vec{k} = 0$$

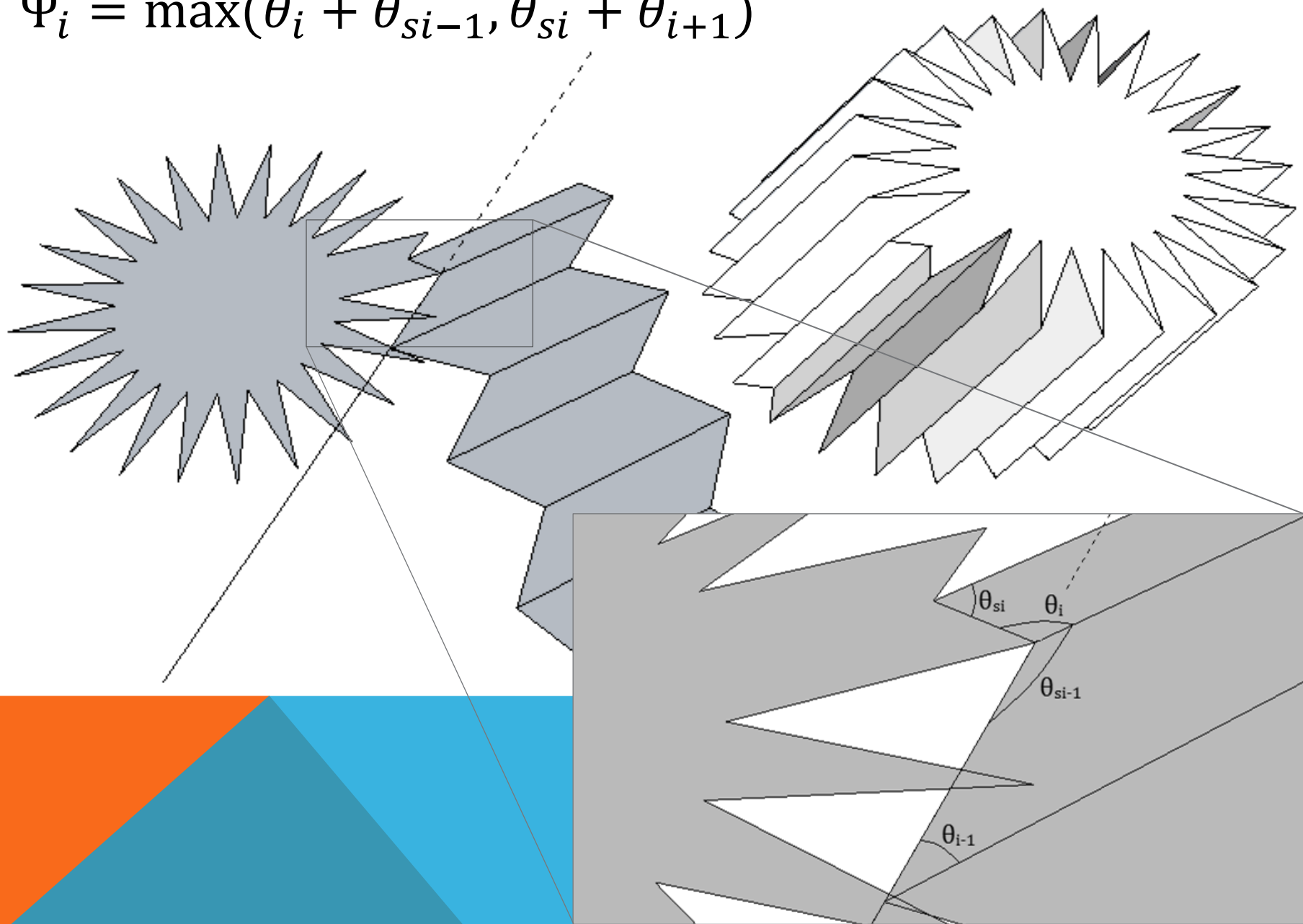
$$\vec{h}' \times \vec{k}' = 0$$

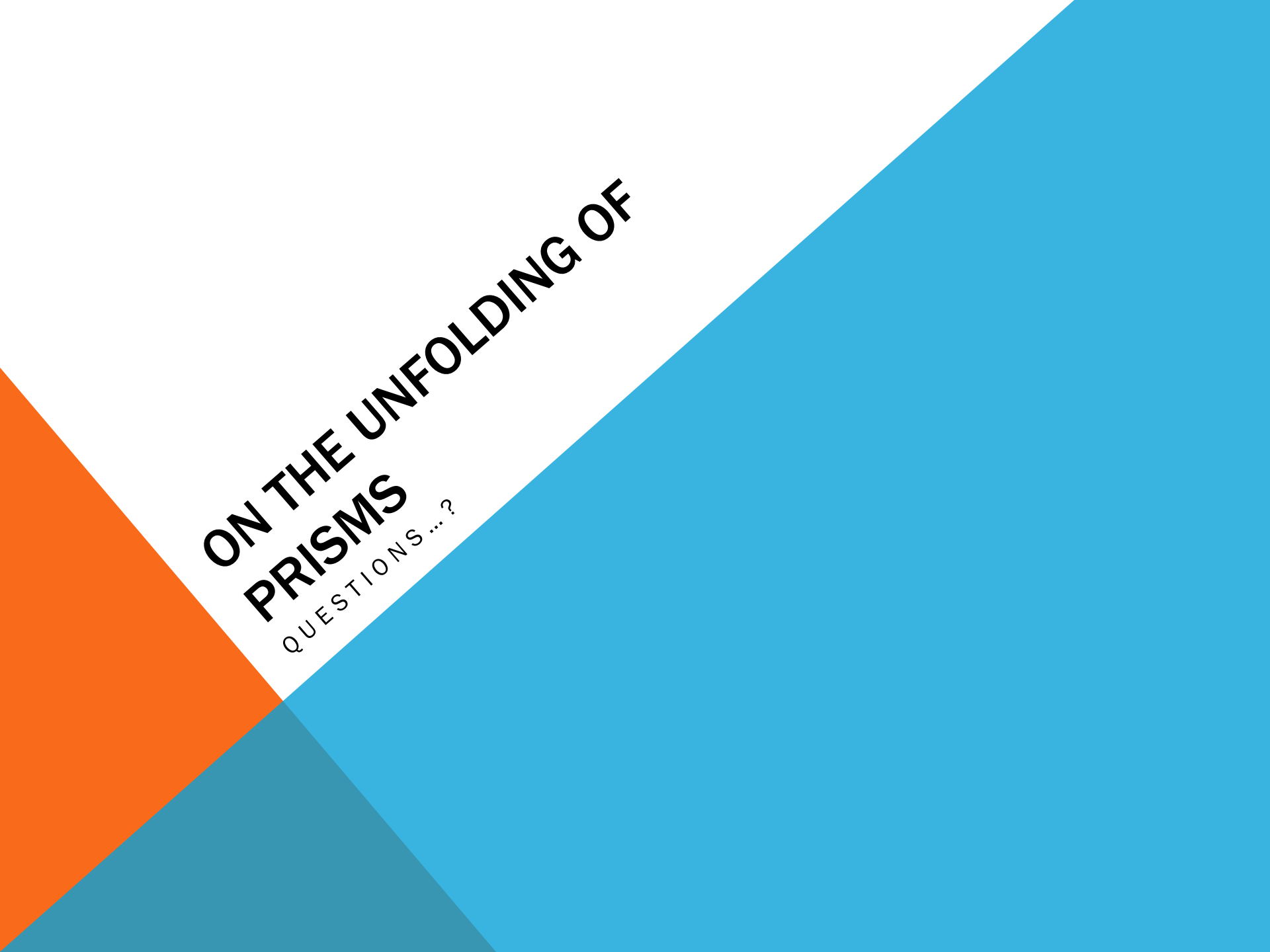




$$\theta = \cos^{-1}(\sin \alpha \sin \beta)$$

$$\Psi_i = \max(\theta_i + \theta_{si-1}, \theta_{si} + \theta_{i+1})$$





**ON THE UNFOLDING OF
PRISMS**

QUESTIONS...?