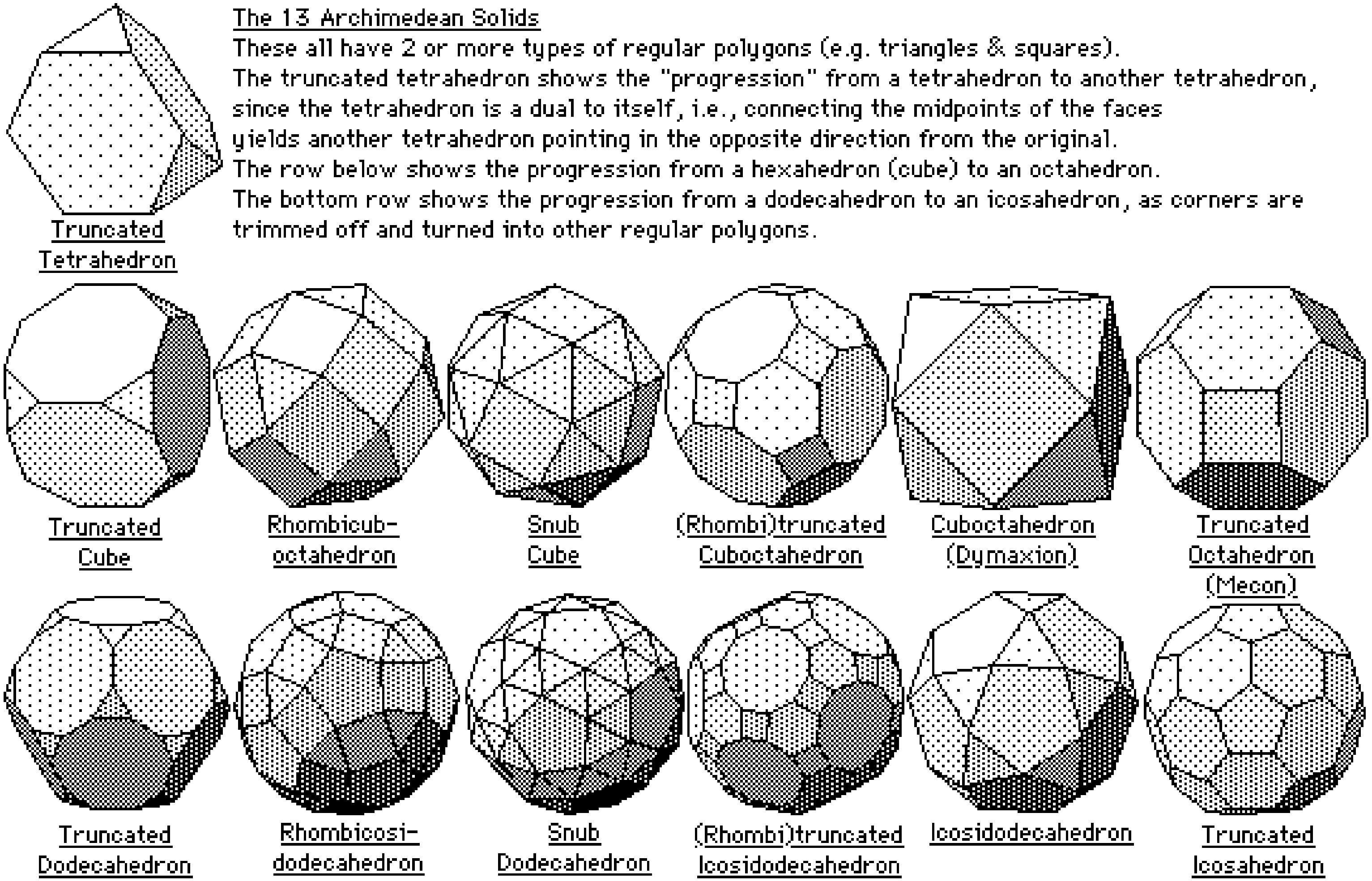
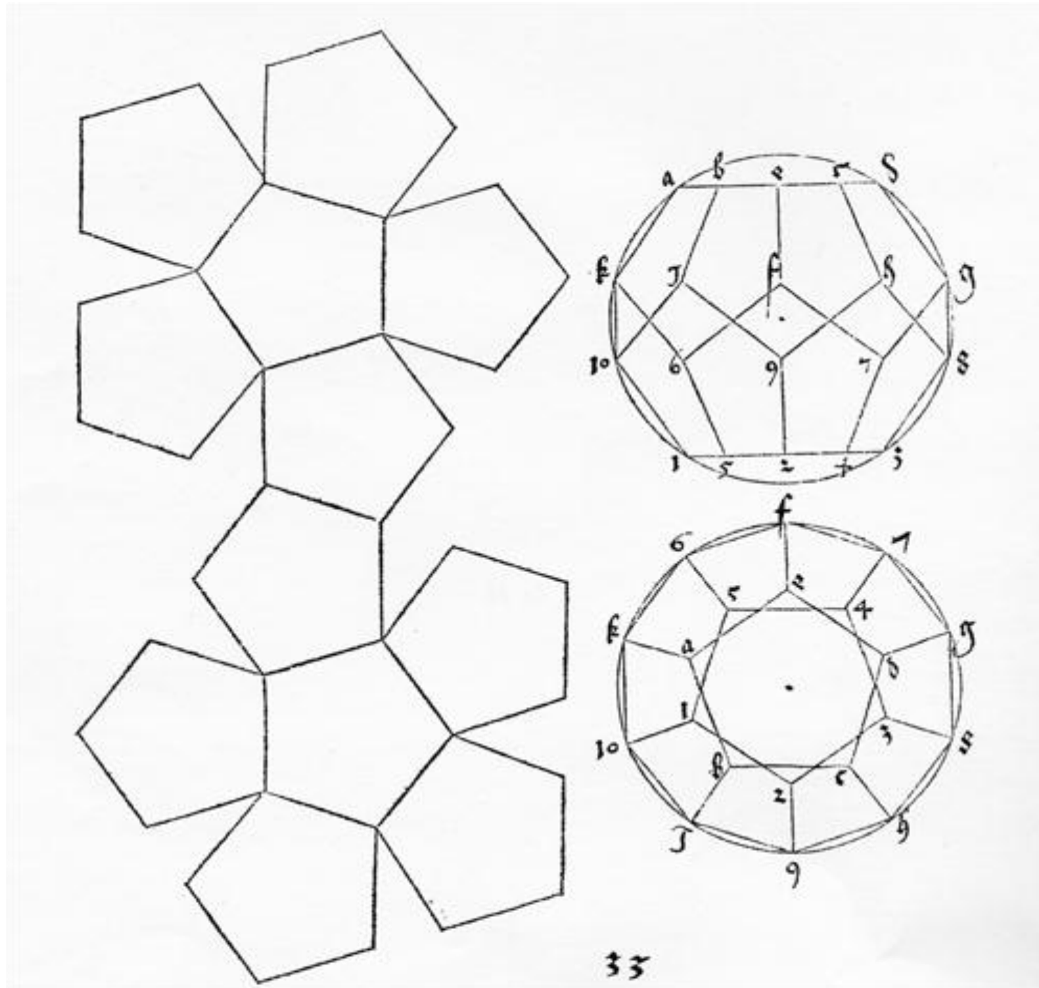
ON THE UNFOLDING OF PRISMS

HONORS THESIS DEFENSE - ANDREW WATSON, ‘12



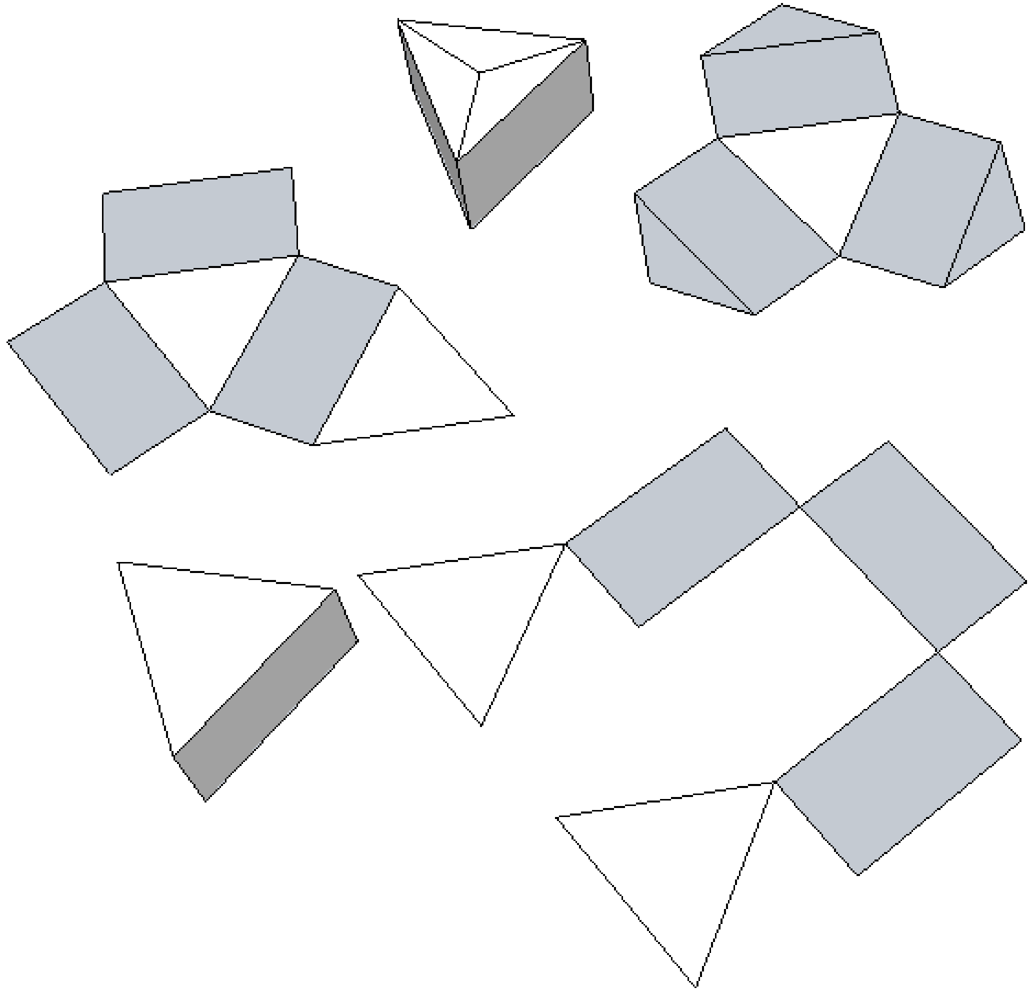
http://bit.ly/HoVeUb

Polygons & Polyhedra



Polyhedral "nets":

i. Non-overlapping
ii. Single polygon
iii. Planar
iv. Cut only along edges of polygons



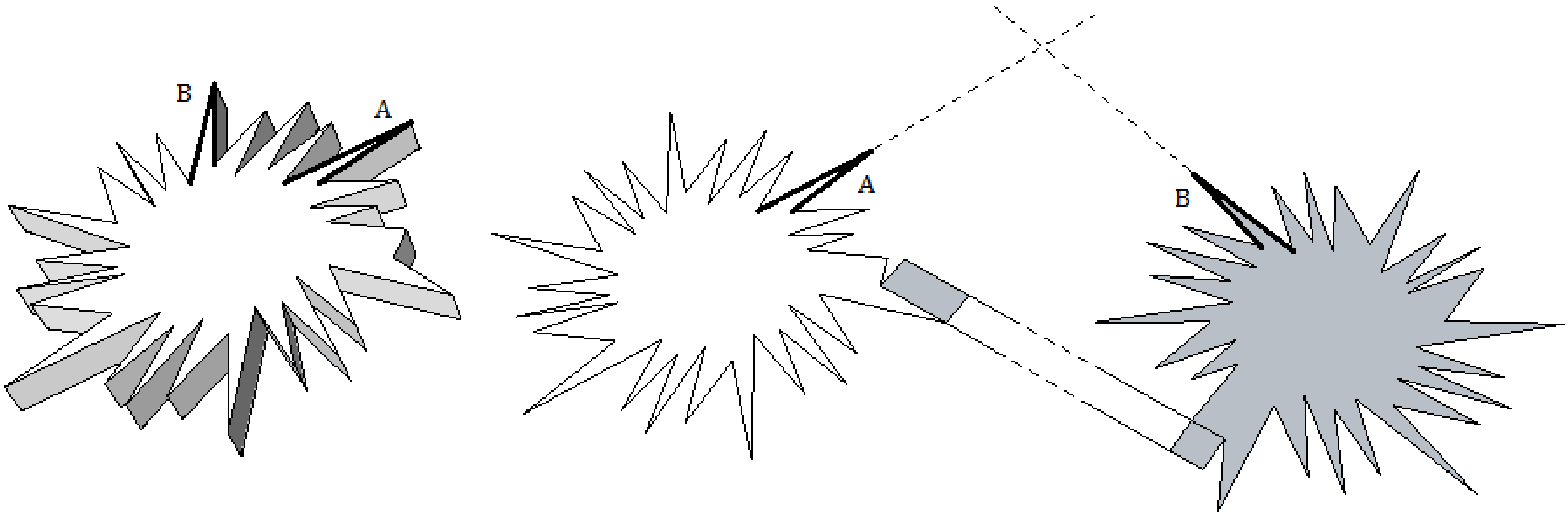
Different “types” of unfoldability:

Edge, **Vertex**, General

<table>
<tr><th colspan="5">Types of Unfoldings for Various Polyhedra</th><th>Edge</th><th>Vertex</th><th>General</th></tr>
<tr><td colspan="5">Triangulated</td><td></td><td>YES</td><td>YES</td></tr>
<tr><td rowspan="13">Non-Triangulated</td><td rowspan="7">Non-Convex</td><td colspan="3"></td><td>NO</td><td></td><td></td></tr>
<tr><td rowspan="4">Prismatoids</td><td>Prismoids, etc.</td><td rowspan="2">Smooth</td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Prisms, Oblique Prisms, Pyramids</td><td>NO</td><td></td><td>YES?</td></tr>
<tr><td rowspan="2">Polygonal</td><td>NO</td><td></td><td>YES?</td></tr>
<tr><td>Prismoids, etc.</td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">Orthogonal Polyhedra</td><td colspan="2"></td><td>NO</td><td>NO</td><td>YES</td></tr>
<tr><td rowspan="2">Orthogonal Terrains</td><td></td><td>YES</td><td>YES</td><td>YES</td></tr>
<tr><td rowspan="6">Convex</td><td>Rectangular Prisms</td><td><u>YES</u></td><td><u>YES</u></td><td><u>YES</u></td></tr>
<tr><td rowspan="4">Prismatoids</td><td>Prismoids, etc.</td><td rowspan="2">Polygonal</td><td></td><td></td><td>YES</td></tr>
<tr><td rowspan="2">Prisms, Oblique Prisms, Pyramids</td><td>YES</td><td>YES</td><td>YES</td></tr>
<tr><td rowspan="2">Smooth</td><td>YES</td><td>YES</td><td>YES</td></tr>
<tr><td>Prismoids, etc.</td><td>YES</td><td>YES</td><td>YES</td></tr>
<tr><td colspan="3"></td><td></td><td></td><td>YES</td></tr>
</table>

Theorem 1:

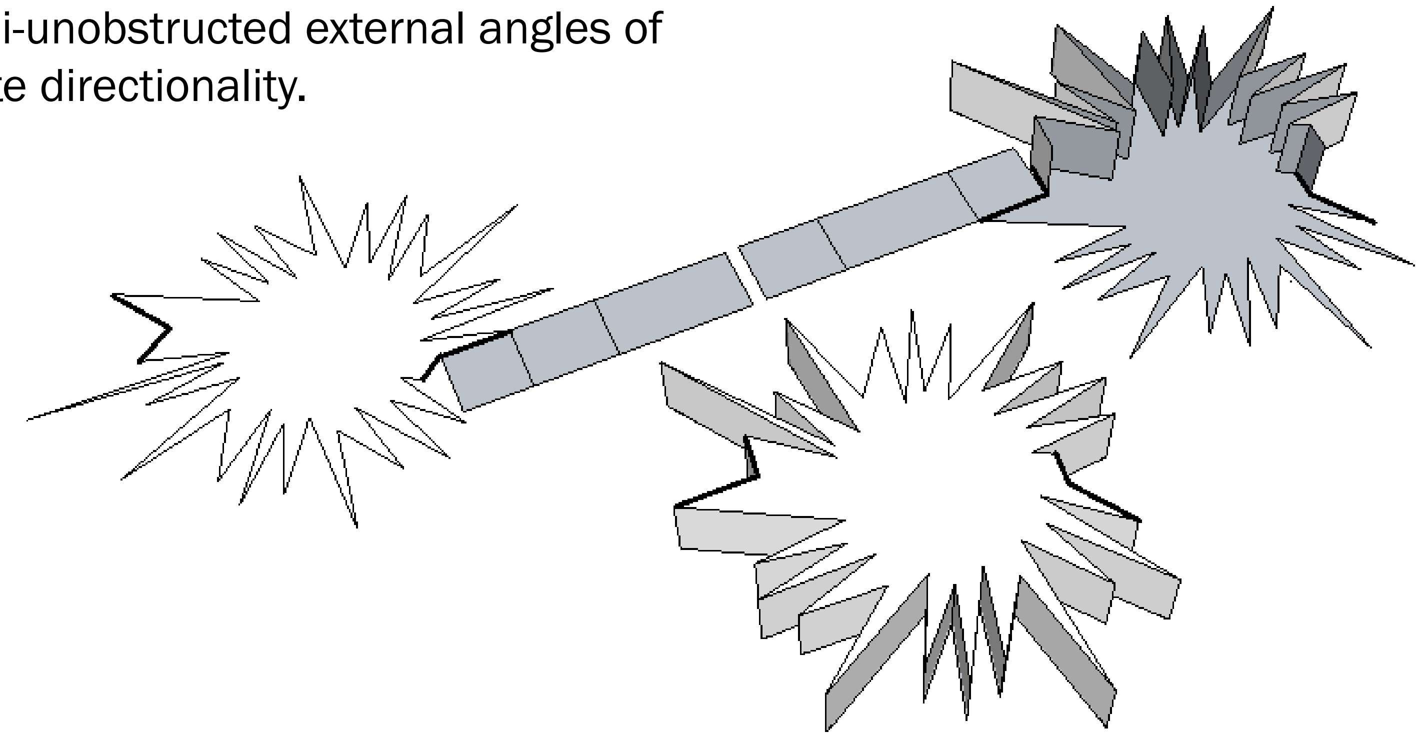
Any genus-0 right prism is unfoldable, given that the base polygon has at least one unobstructed external angle of $\geq 90°$.

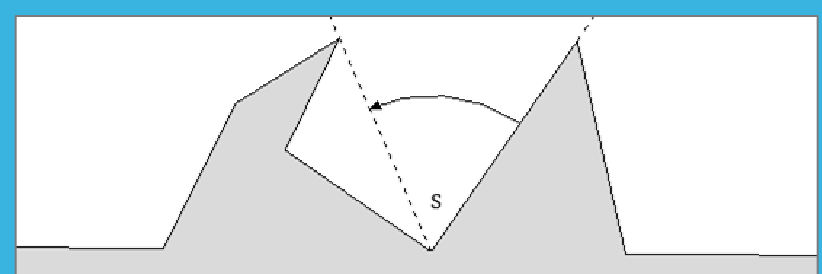


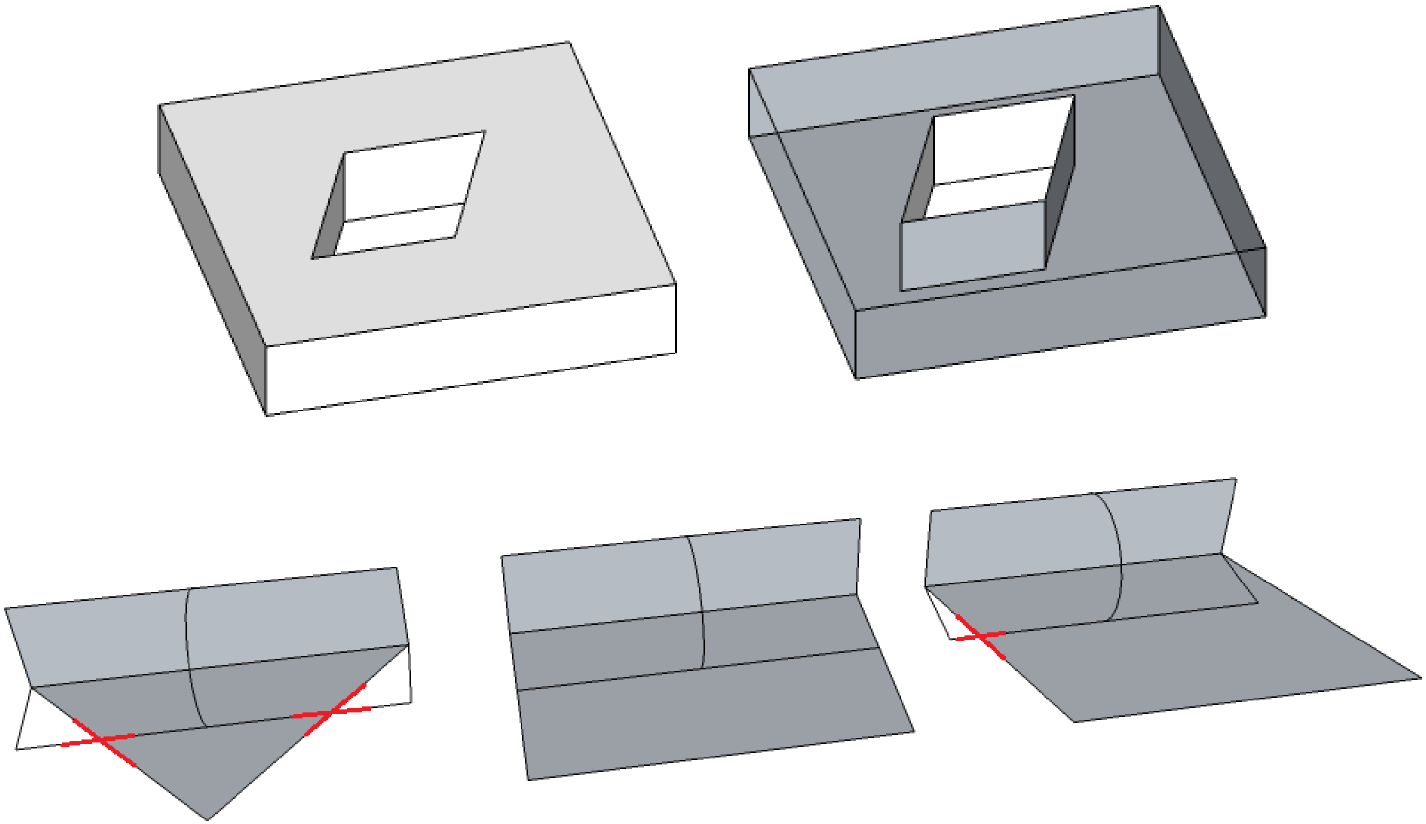
Note: Any spikes A & B, will not crash due to perimeters

Corollary 1-1:

A sufficiently short prism is partially unfoldable if its base has two semi-unobstructed external angles of ≥90° with opposite directionality.



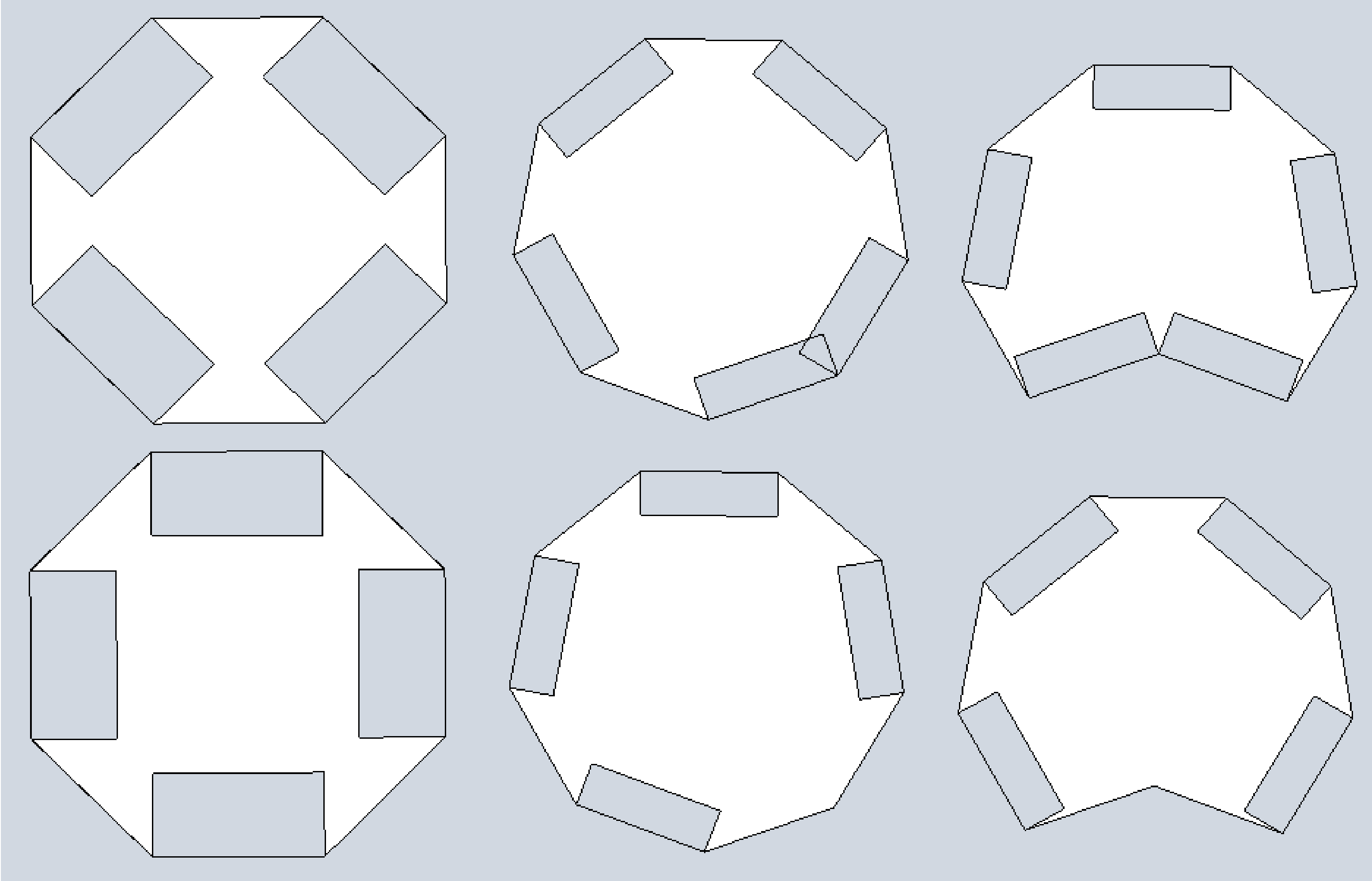




Genus, and how we apply it to polygons

Important characteristics of holes:

i. Angles of holes
ii. Number of sides of holes



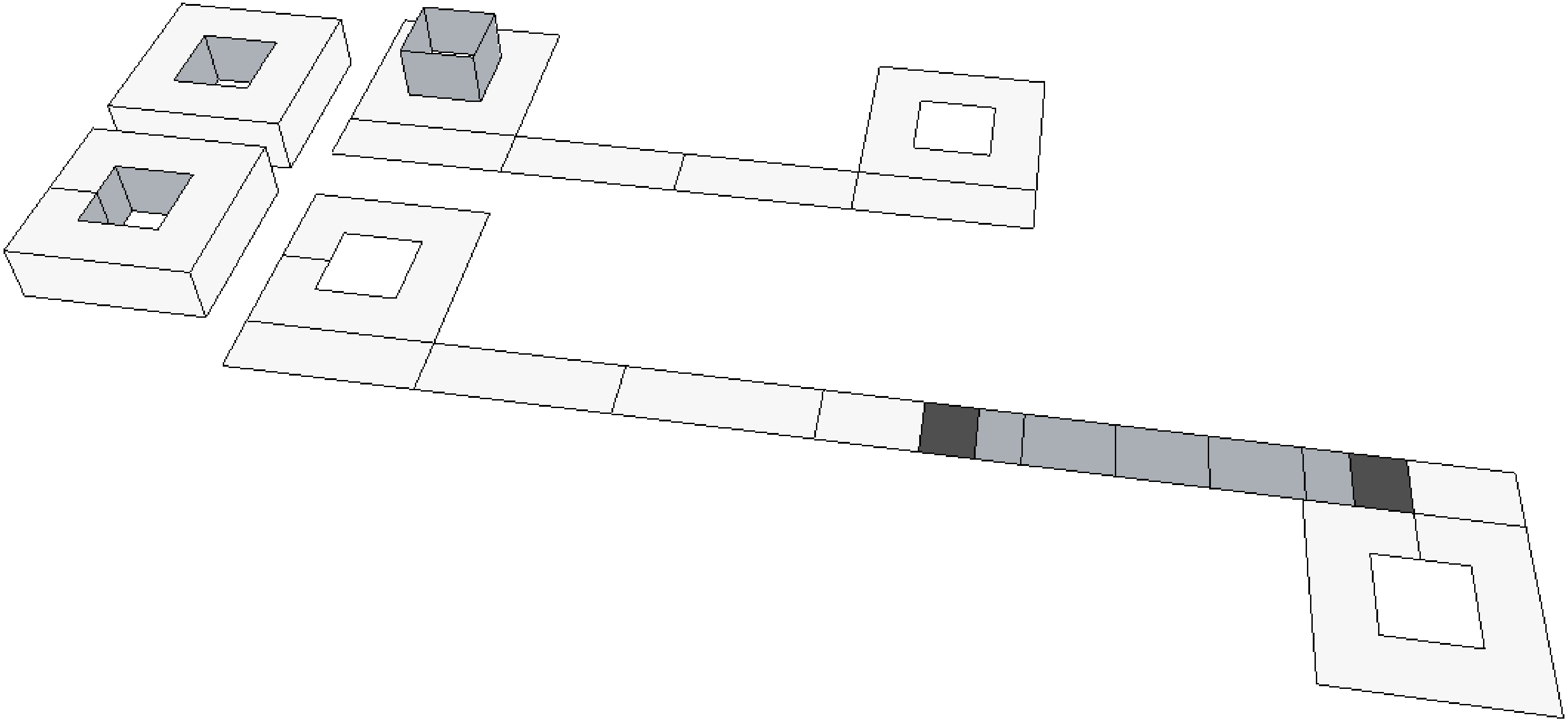
Theorem 2:

Let P be a properly unfoldable genus-1+ prism of height ϵ. P is edge-unfoldable for sufficiently small ϵ if and only if:

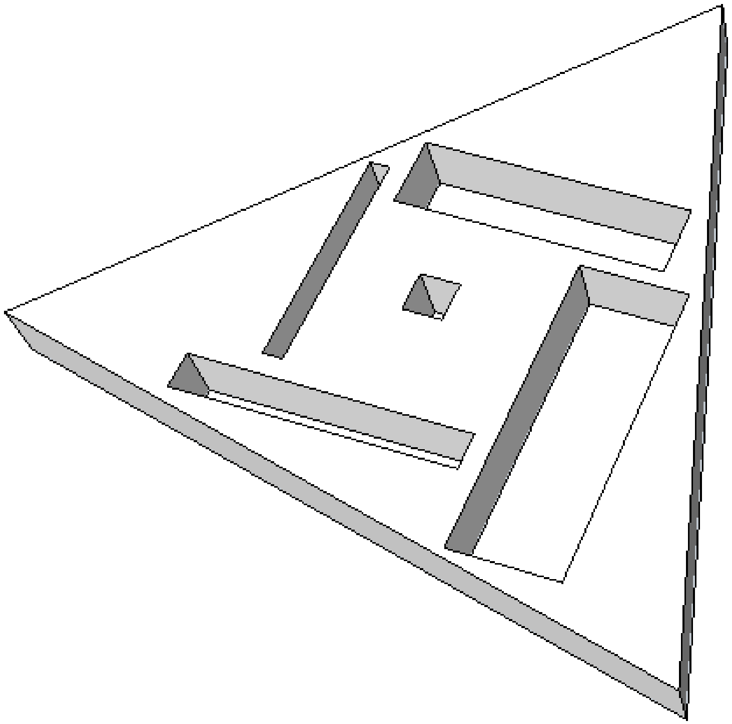
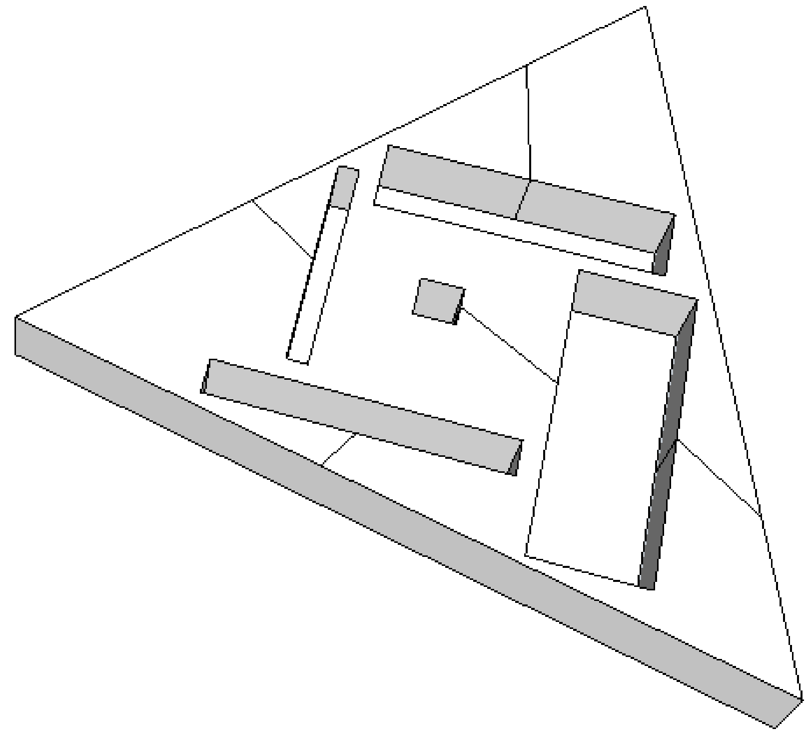
1. All exterior angles of all holes are ≥90°
2. All even-edged holes have ≥4 edges
3. All odd-edged holes have ≥ 5 edges
4. All odd-edged holes have at least one exterior angle of >180°

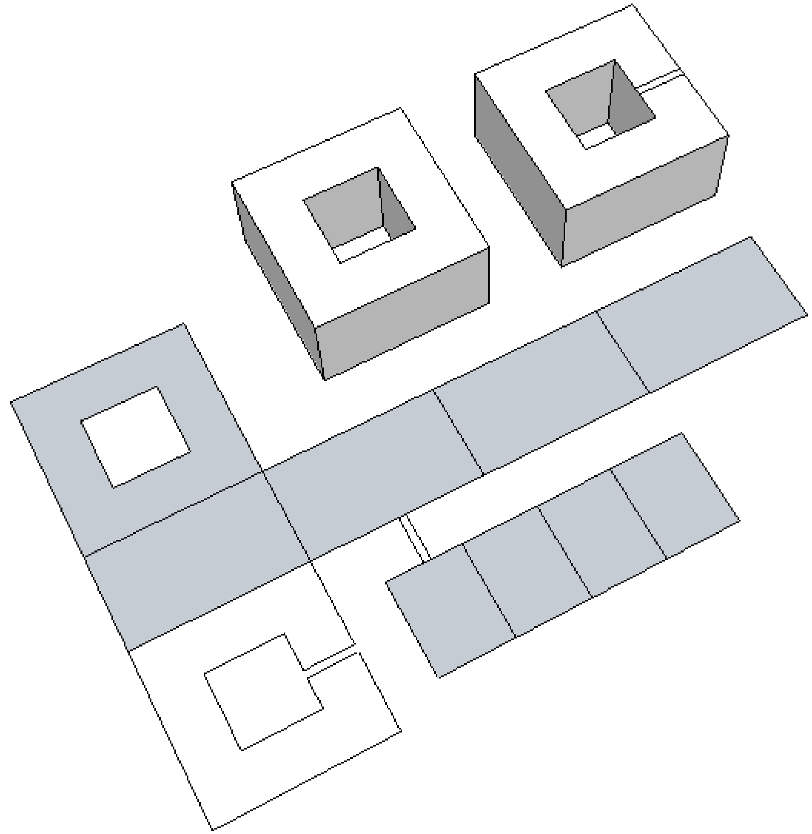
…but what if a prism isn't unfoldable?

Can we "cut" it (add edges) to make if unfoldable?



…making a prism unfoldable by adding faces

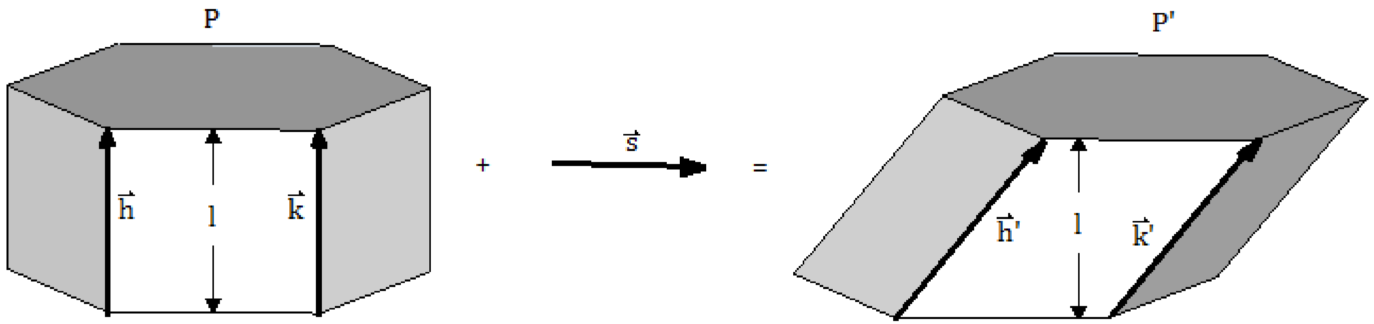





...making a prism unfoldable by adding edges

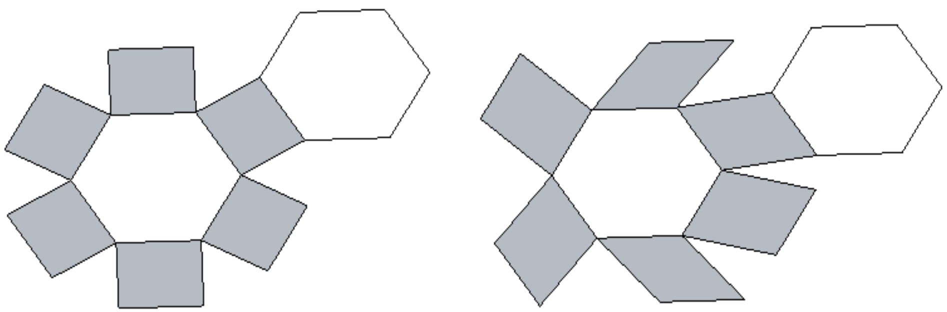
Relationships between faces are preserved, no matter where the edges are cut...

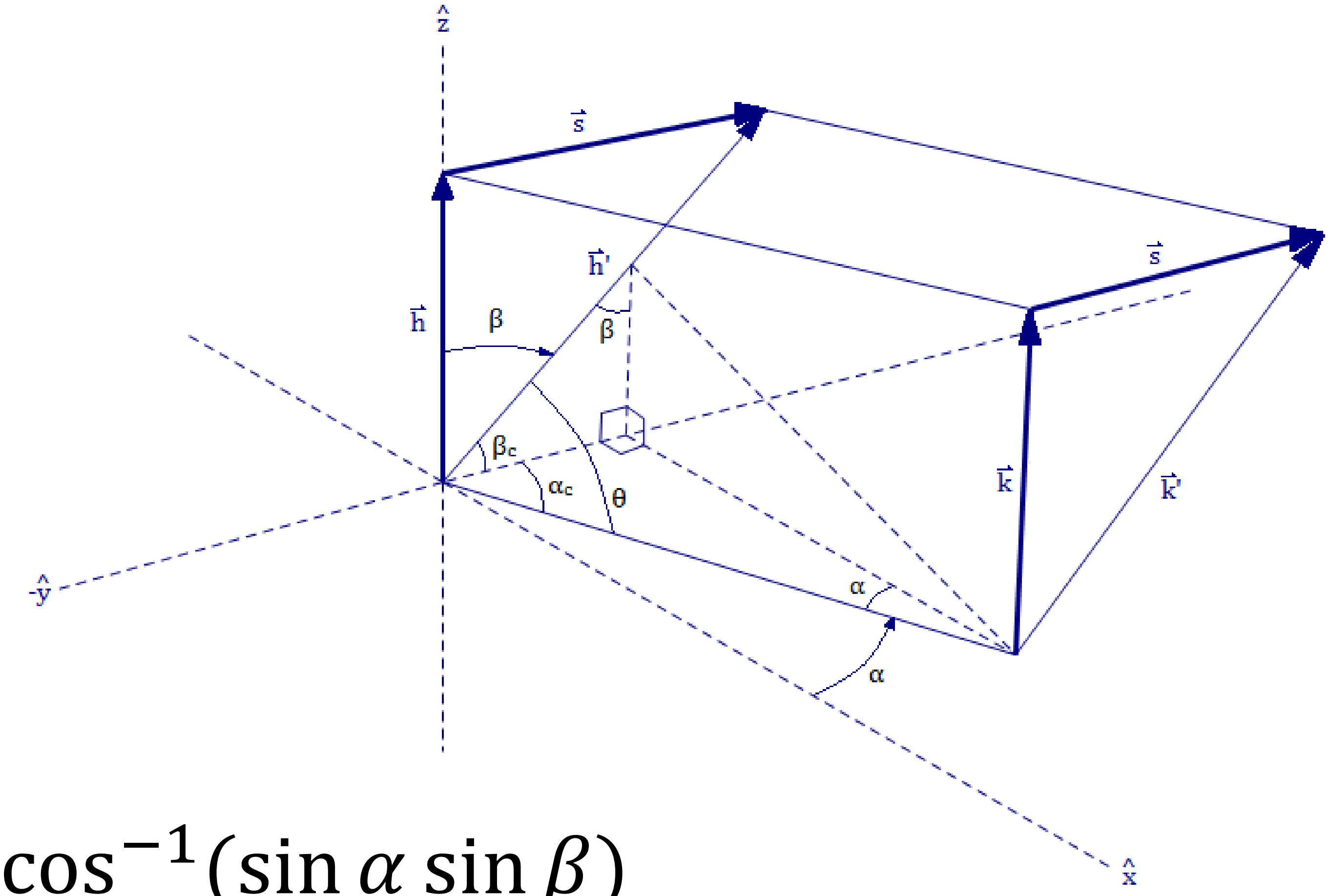
...therefore, there is a finite number of unfoldings!



$$\vec{h} \cdot \vec{k} = l^2 \qquad \vec{h}' \cdot \vec{k}' = l^2 + |\vec{s}|^2$$

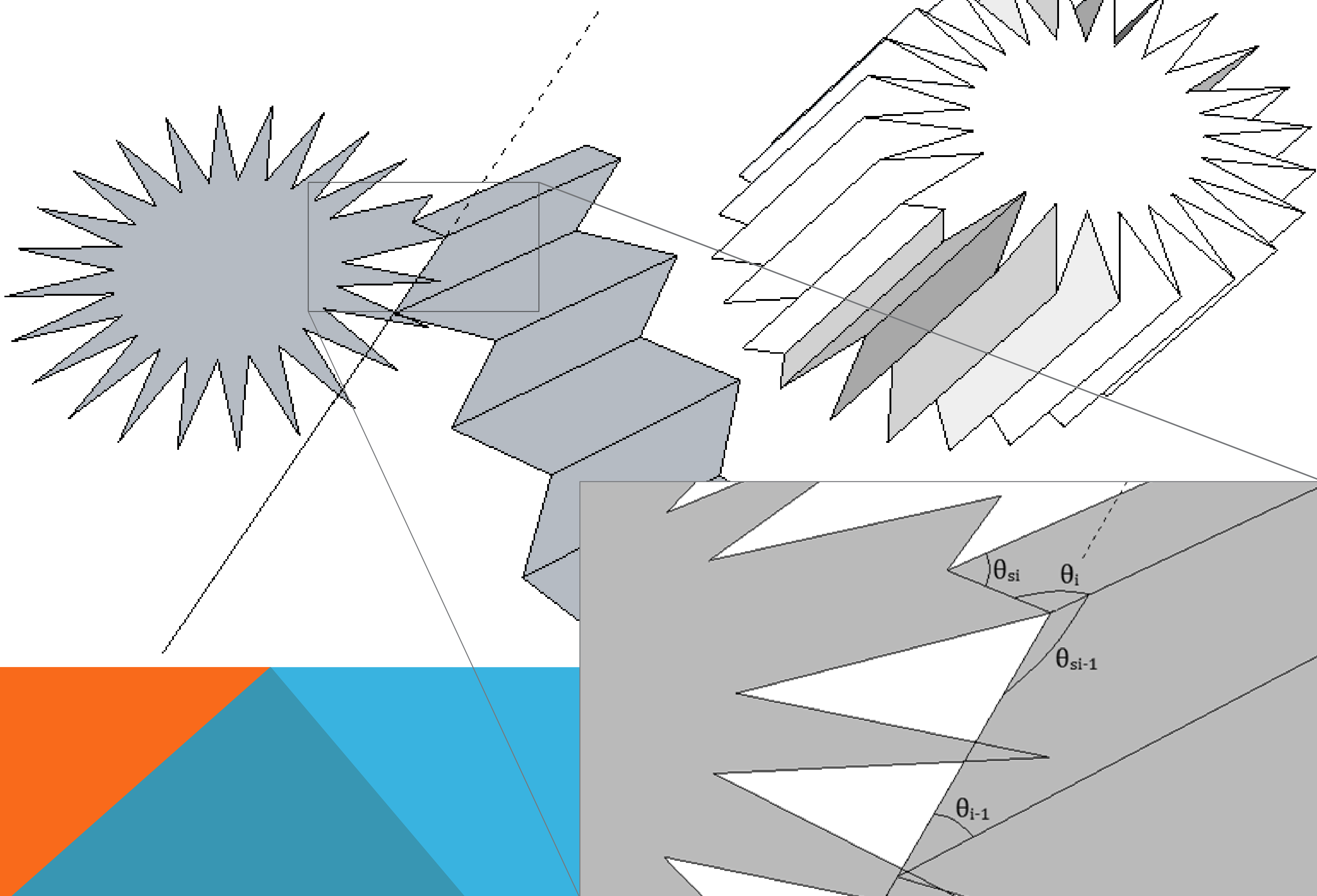
$$\vec{h} \times \vec{k} = 0 \qquad \vec{h}' \times \vec{k}' = 0$$





$$\theta = \cos^{-1}(\sin\alpha \sin\beta)$$

$$\Psi_i = \max(\theta_i + \theta_{si-1}, \theta_{si} + \theta_{i+1})$$



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Questions…?